University of Al-Hamdaniya
Computer Science Department
Digital image processing

Filters in Frequency Domain

Lec-13

By

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Image Enhancement in Frequency Domain

Image processing in the frequency domain

- Image smoothing
- Image sharpening

There are basically three different kinds of filters :

1. Low-pass filtering.
2. High-pass filtering
3. Band-pass filter.

Image Enhancement in Frequency Domain

1. Low-pass filtering

- **Edges and sharp transitions** in the gray levels of an image contribute significantly to the high-frequency content of its Fourier transform
- **Blurring (smoothing) is achieved** in the frequency domain by attenuating a specified rage of high-frequency components.

يقوم المرشح باستبعاد القيم ذات الترددات العالية والإبقاء على القيم ذات الترددات الواطئة وبالتالي نحصل على تنعيم الصورة

This task is performed through low-pass filtering

There are three types of low-pass filters :

a. Ideal low-pass filter (very sharp).
b. Butterworth low-pass filter (tunable)
c. Gaussian low-pass filter (very smooth).

Image Enhancement in Frequency Domain

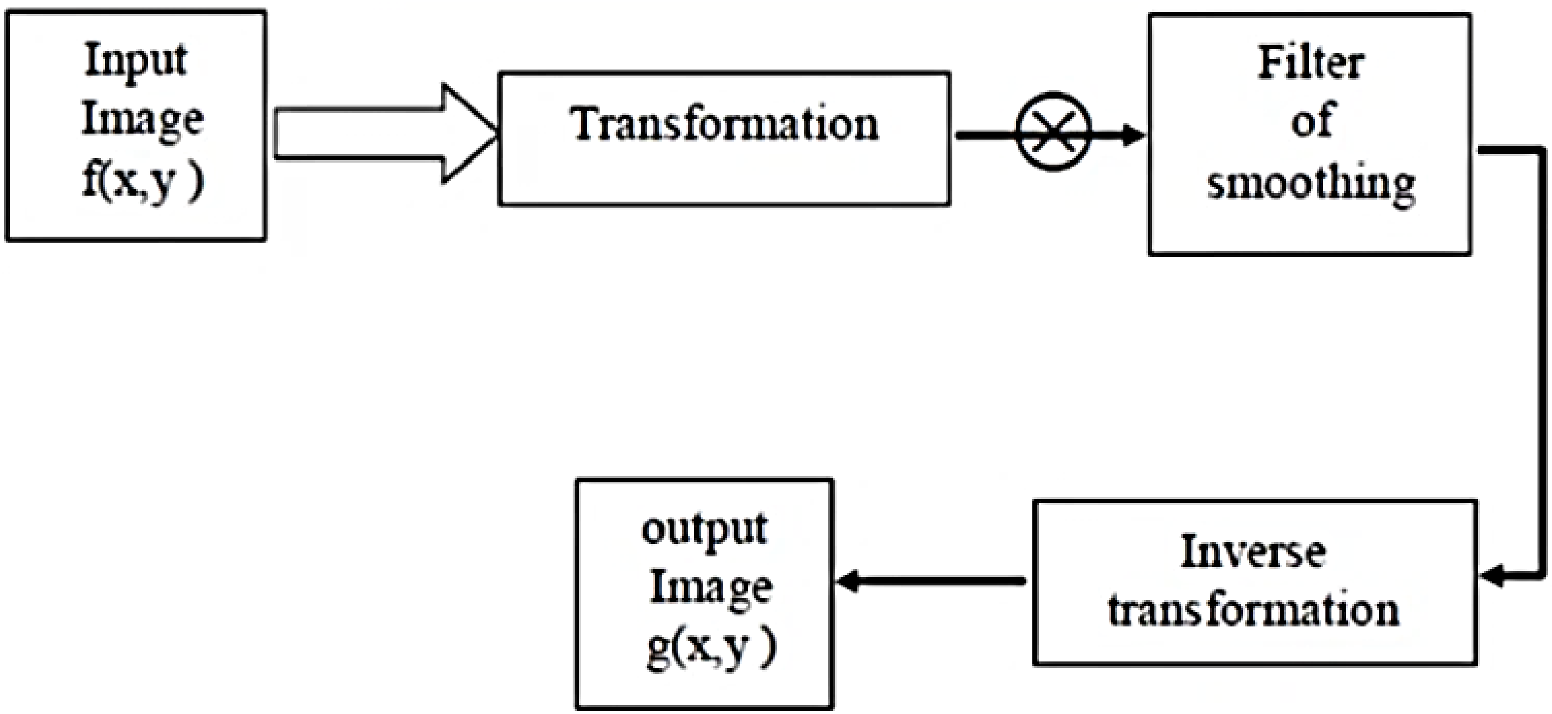


Image Enhancement in Frequency Domain

a. Ideal low-pass filter

- The ideal low-pass filter is one which satisfies the relation:

$$H(u,v) = \begin{cases} 1, & if \quad D(u,v) \le D_0 \\ 0, & if \quad D(u,v) > D_0 \end{cases}$$

- Where D_0 is a specified nonnegative quantity, and $D(u,v)$ is the distance from point (u,v) To the origin of the frequency plane;

$$D(u,v) = \sqrt{u^2 + v^2}$$

- The filter is called ideal because all the frequencies inside the circle of radius D_0 are passed with no attenuation تخفيف, whereas all frequencies outside this circle are completely attenuated.

Image Enhancement in Frequency Domain

a. Ideal low-pass filter

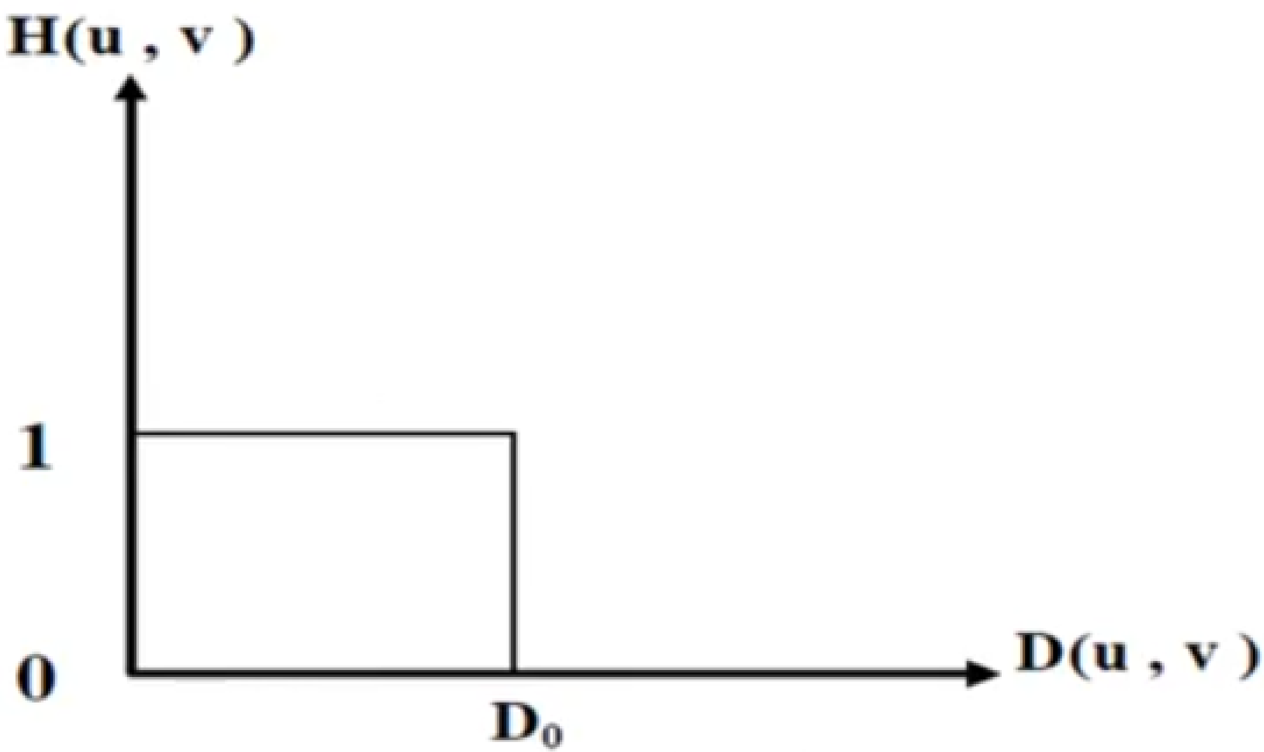


Image Enhancement in Frequency Domain

a. Ideal low-pass filter

Original Image | LPF image, $D0$ =57 | LPF image, $D0$ =36 | LPF image, $D0$ =26

Image Enhancement in Frequency Domain

b. Butterworth low-pass filter

➢The **Butterworth low-pass** filter is an approximation to the **ideal filter without the step discontinuity**. The transfer function of the **Butterworth low-pass** filter of order n and with **cut-off frequency locus at a distance D_0** from the origin is defined by the relation:

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)}{D_0}\right]^{2n}}$$

Where $D(u,v)$ is given by,

$$D(u,v) = \sqrt{u^2 + v^2}$$

Unlike the ideal low-pass filter, the Butterworth filter **does not have a sharp discontinuity** that establishes a clear **cut-off between passed and filtered frequencies**.

Image Enhancement in Frequency Domain

b. Butterworth low-pass filter

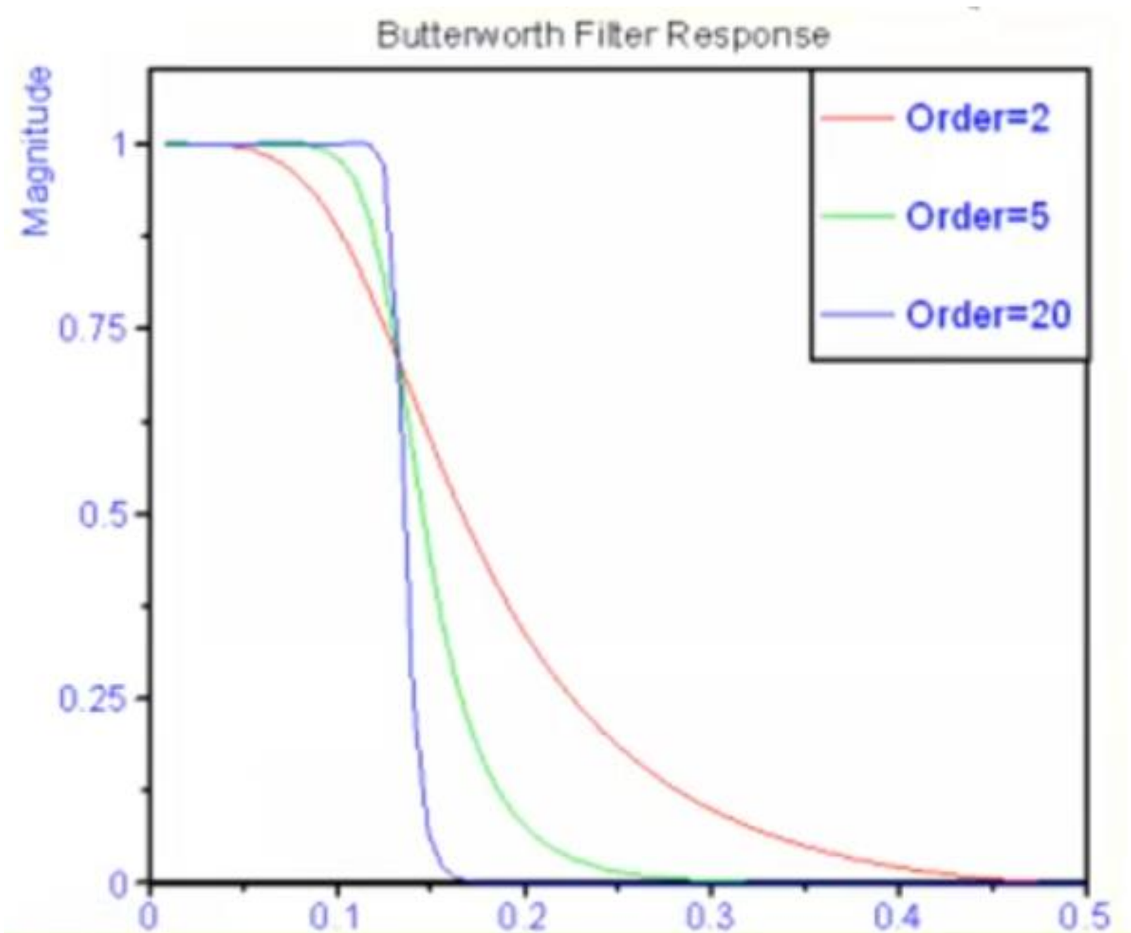


Image Enhancement in Frequency Domain

b. Butterworth low-pass filter

Original Image | LPF image, $D0 = 18$ | LPF image, $D0 = 13$ | LPF image, $D0 = 10$

c. Gaussian low-pass filter:

➢The form of a Gaussian low pass filter GLPF in 2D is:

$$H(u,v) = e^{\frac{-D^2(u,v)}{2\sigma^2}}$$

- Where $D(u,v)$ is given by.

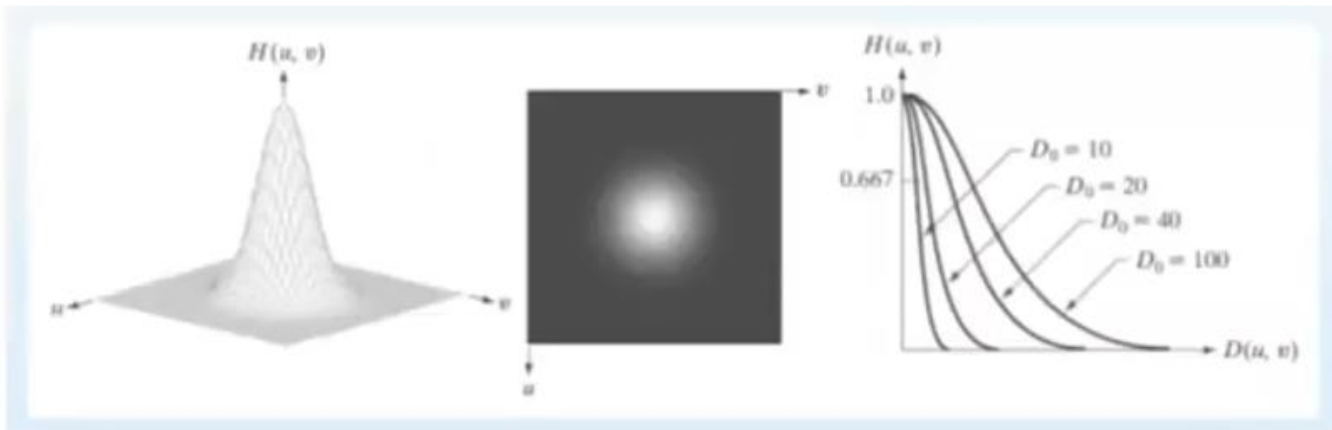
$$D(u,v) = \sqrt{u^2 + v^2}$$

- σ: mesure of the spread of the Gaussian curve
- Let σ = D0, then:

$$H(u,v) = e^{\frac{-D^2(u,v)}{2D_0^2}}$$

Image Enhancement in Frequency Domain

c. Gaussian Low-pass filter



Original image

Result of filtering with Gaussian filter with cutoff radius 5

Result of filtering with Gaussian filter with cutoff radius 15

Result of filtering with Gaussian filter with cutoff radius 30

Result of filtering with Gaussian filter with cutoff radius 85

Result of filtering with Gaussian filter with cutoff radius 230

Image Enhancement in Frequency Domain

- **2. High-pass filtering:**
- Image sharpening can be achieved in the frequency domain by a high-pass filtering process. High-pass filter will attenuate the low-frequency components without disturbing high frequency information.
- في هذا النوع من الترشيح فان الترددات العالية تمر (تترشح) وتبقى الترددات الواطئة (تستبعد) بمعنى آخر، يقوم المرشح بإبعاد القيم ذات الترددات الواطئة و الإبقاء على القيم ذات الترددات العالية (أي بمعنى آخر، يقوم بزيادة حدة الصورة لان القيم العالية تمثل الحواف والتشويه)

- **The high-pass filters that we will consider are:**
- a. Ideal high-pass filter and
- b. Butterworth high-pass filter
- c. Gaussian high-pass filter

Image Enhancement in Frequency Domain

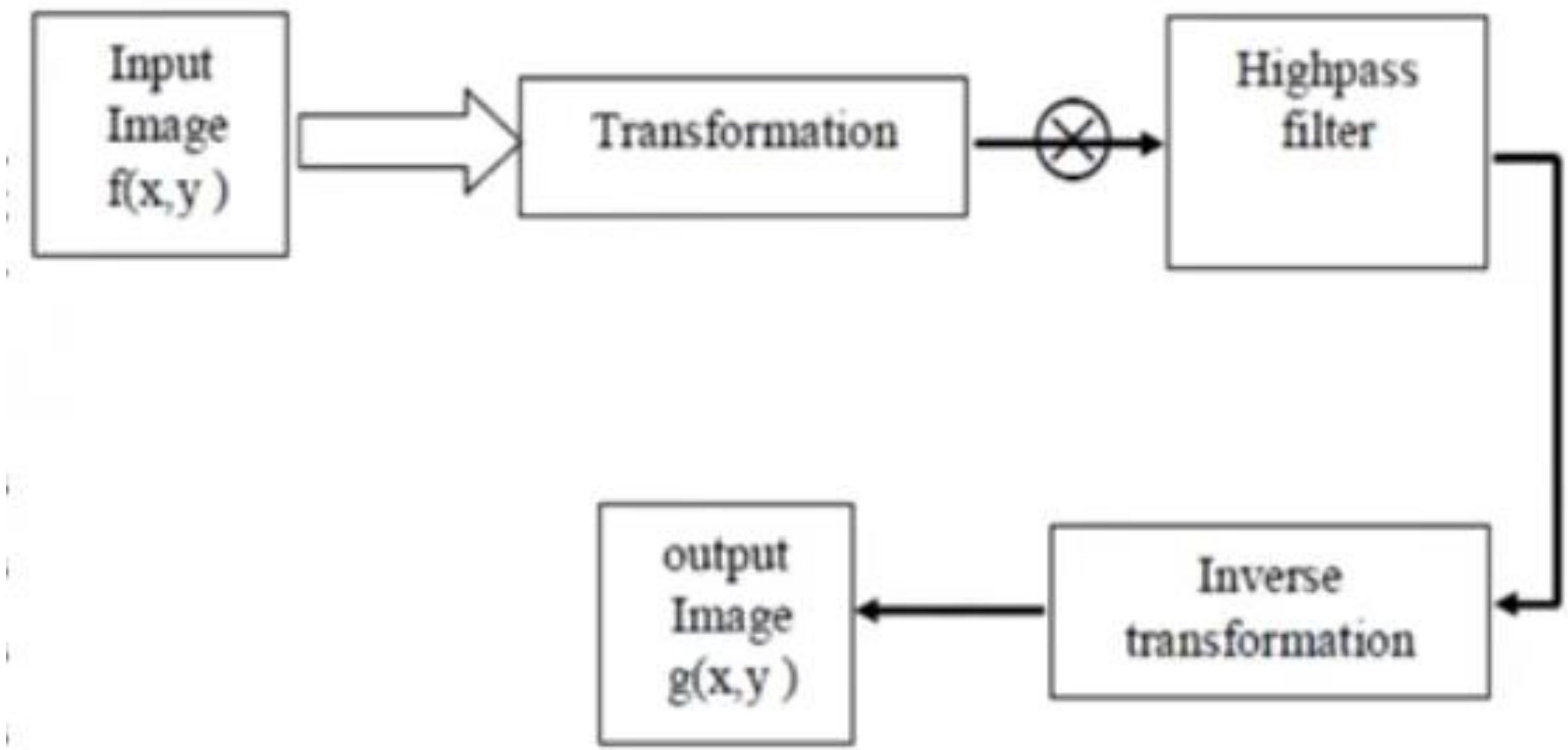


Image Enhancement in Frequency Domain

a. Ideal High-pass filter

- The high-pass filter is obtained from a given lowpass filter using :

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

A 2-D ideal highpass filter (IHPL) is defined as

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

- Where D_0 is a specified nonnegative quantity, and$D\ (u,v)$ is the distance from point (u,v) To the origin of the frequency plane;

$$D(u,v) = \sqrt{u^2 + v^2}$$

- This filter completely attenuates all frequencies inside a circle of radius D_0 while passing, **without attenuation**, all frequencies outside the circle.

Image Enhancement in Frequency Domain

a. Ideal High-pass filter

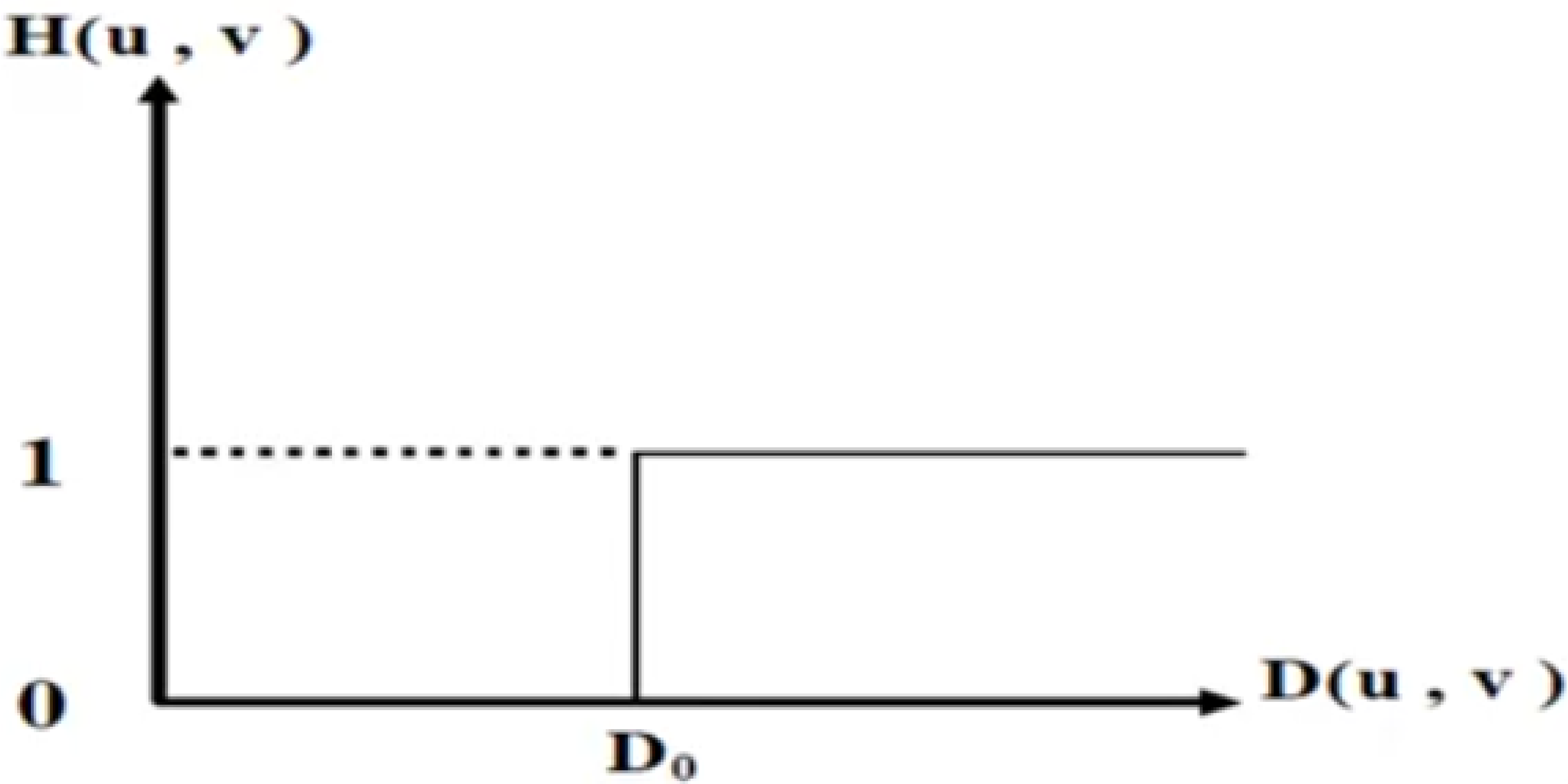
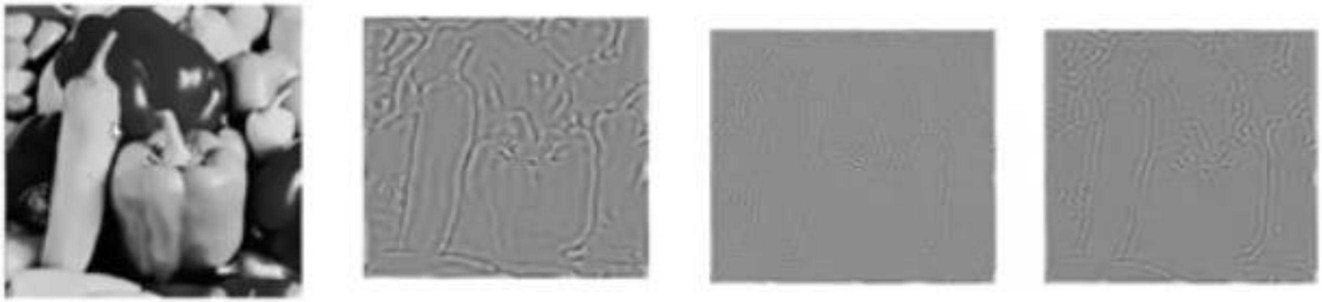
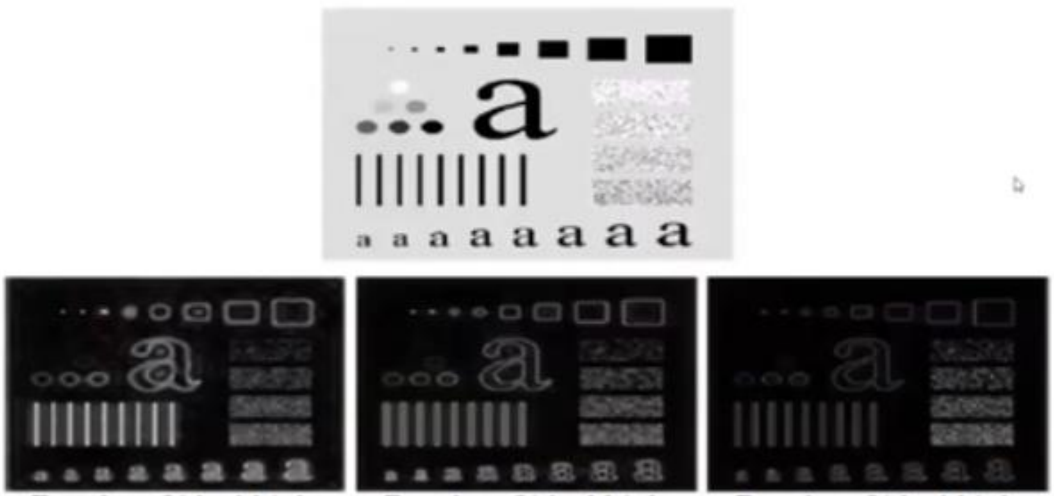


Image Enhancement in Frequency Domain

a. Ideal High-pass filter



Original Image LPF image, $D0$ =13 LPF image, $D0$ =36 LPF image, $D0$ =26



Results of ideal high pass filtering with D_0 = 15

Results of ideal high pass filtering with D_0 = 30

Results of ideal high pass filtering with D_0 = 80

Image Enhancement in Frequency Domain

a. Butterworth High-pass filter

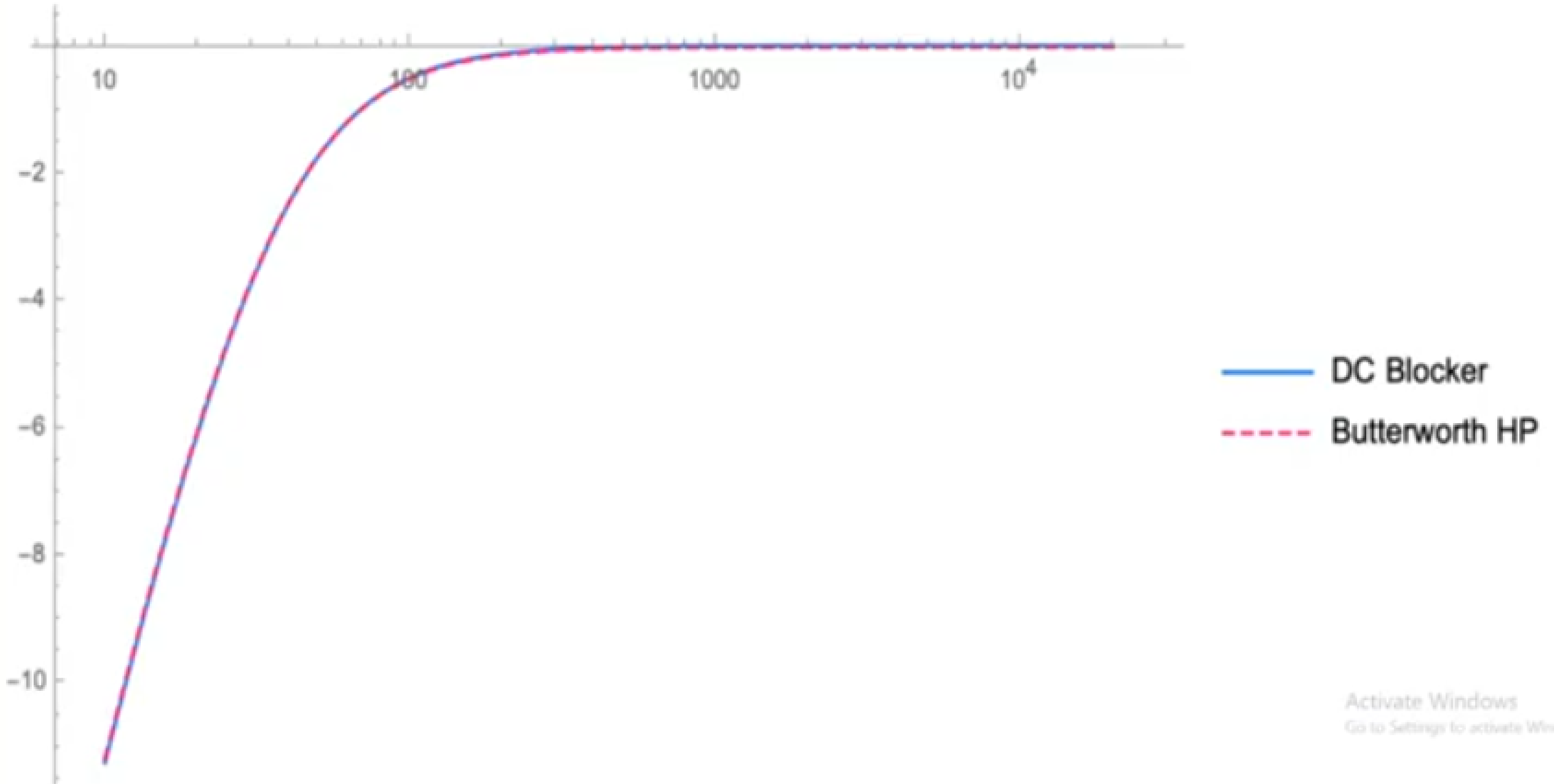
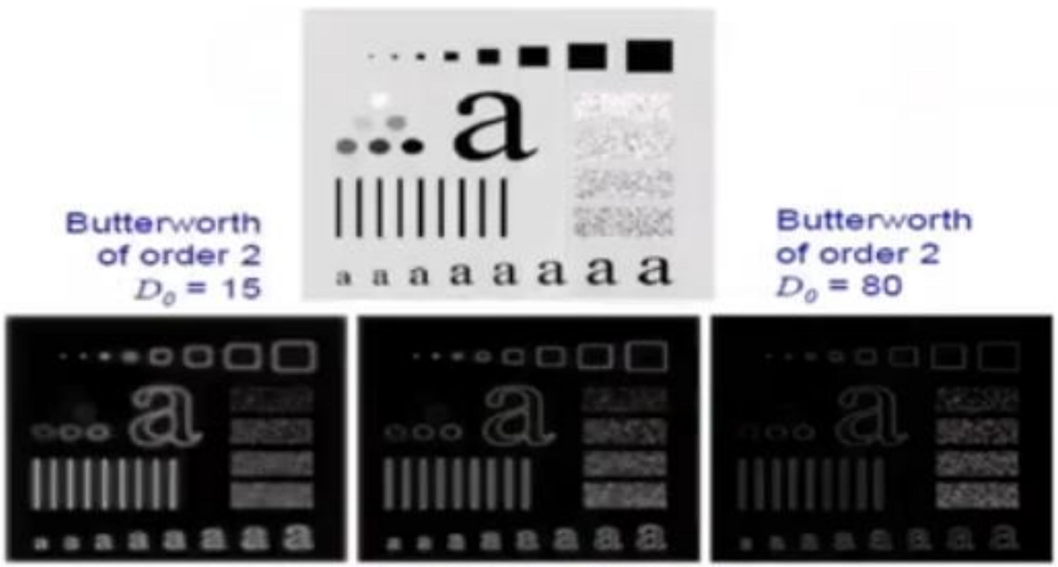


Image Enhancement in Frequency Domain

b. Butterworth High-pass filter

Original Image | LPF image, $D0$ =47 | LPF image, $D0$ =36 | LPF image, $D0$ =81



Butterworth of order 2 D_0 = 15

Butterworth of order 2 D_0 = 80

Butterworth of order 2 D_0 = 30

Image Enhancement in Frequency Domain

c. Gaussian High-pass filter

➢The form of a Gaussian high pass filter GHPF in 2D is:

$$H(u,v) = e^{\frac{-D^2(u,v)}{2\sigma^2}}$$

- Where $D(u,v)$ is given by.

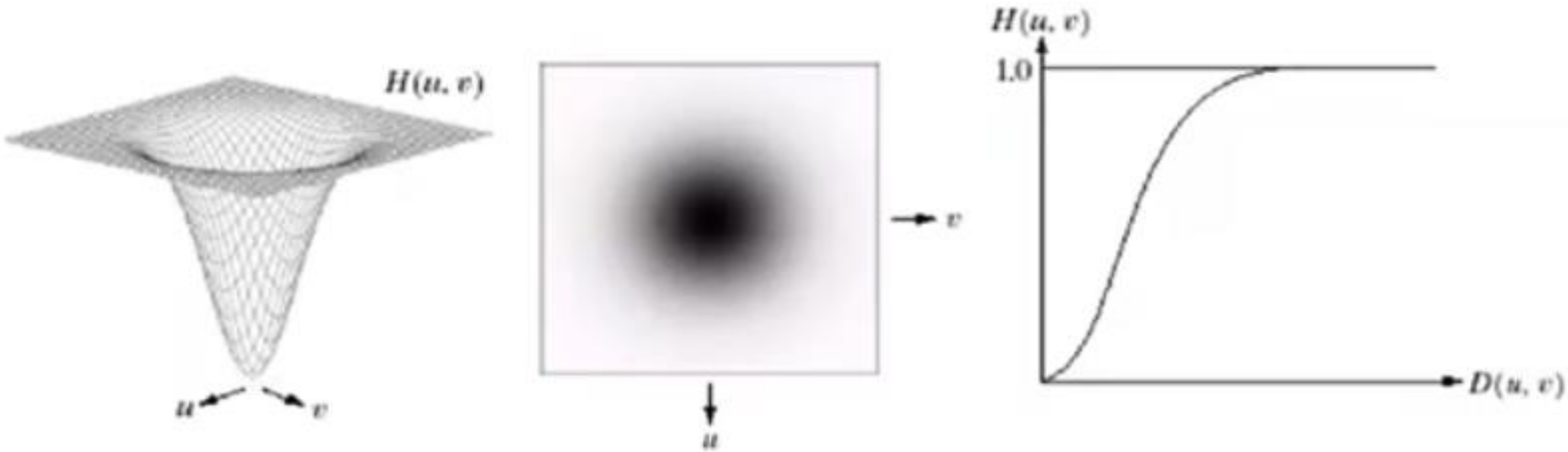
$$D(u,v) = \sqrt{u^2 + v^2}$$

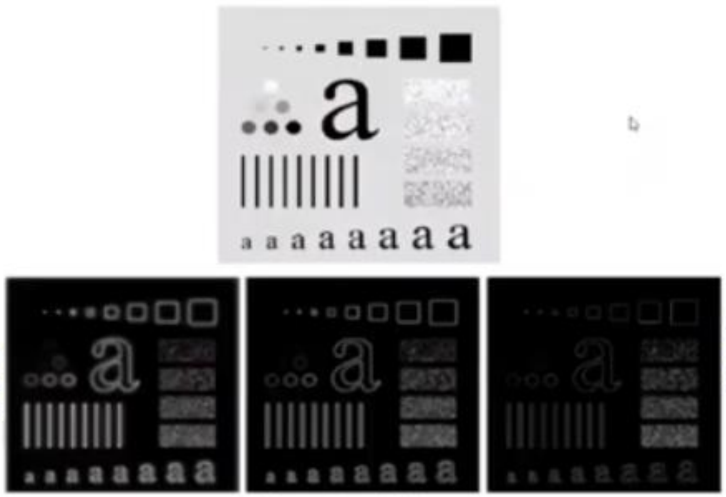
- σ: mesure of the spread of the Gaussian curve
- Let σ = D0, then:

$$H(u,v) = e^{\frac{-D^2(u,v)}{2D_0^{\,2}}}$$

Image Enhancement in Frequency Domain

c. Gaussian High-pass filter





Gaussian high pass D_0 = 15

Gaussian high pass D_0 = 30

Gaussiar high pass D_0 = 80

Image Enhancement in Frequency Domain

The difference between The frequency domain and spatial domain

1. The result form frequency domain obtain by Fourier transform **while** result from the spatial domain from sampling and quantization.
2. The **frequency domain** refer to the space defined by values of the Fourier transform and it is frequency variables (u,v), **while** the **spatial domain** refer to the image plane itself, the total number of pixel composing an image, each has spatial coordinates (x,y).
3. The frequency domain has **complex** quantities **while** the spatial domain has **integer** quantities.

End of Lecture