University of Al-Hamdaniya
Computer Science Department
Digital image processing

Image Enhancement in Frequency Domain

Lec-12

By

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Image Enhancement in Frequency Domain

- The **principal objective of enhancement** is to process a given image so that the result is more **suitable than the original image** for a specific application .
- **A frequency content** refers to the rate at which the gray level change in the imager, where **the rapidly changing in brightness values correspond to high frequency**, while the **slowly change in brightness values correspond to low frequency**.
- The **convolution theorem** is the foundation of **frequency domain** techniques. Consider the following **spatial domain** operations :

$$g(x,y) = h(x,y) * f(x,y)$$

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- The convolution theorem states that following frequency domain relationship holds:

$$G(u,v) = H(u,v)\ F(u,v)$$

- Where G, H and F are the Fourier transforms of ***g*, *h*** and ***f*** respectively.
- H is known as the transfer function of the process. The goal is to select a transfer function that changes the image in such a way that some feature of the image is enhanced.
- Examples include **edge detection, noise removal, emphasis of information is the image.**

Image Enhancement in Frequency Domain

Basics of filtering in the frequency domain

The frequency filters process an image in the frequency domain may need the following steps:

1. **Transform the image f(x,y) into the Fourier domain**
   Compute **F(u, v)** it is the **DFT** of the image

2. **Multiply F(u,v) of the image by the filter.**
   i.e. multiply **F(u, v)** by a filter function **H(u,v)**.

3. **Find the inverse transform of the image G(u,v).**

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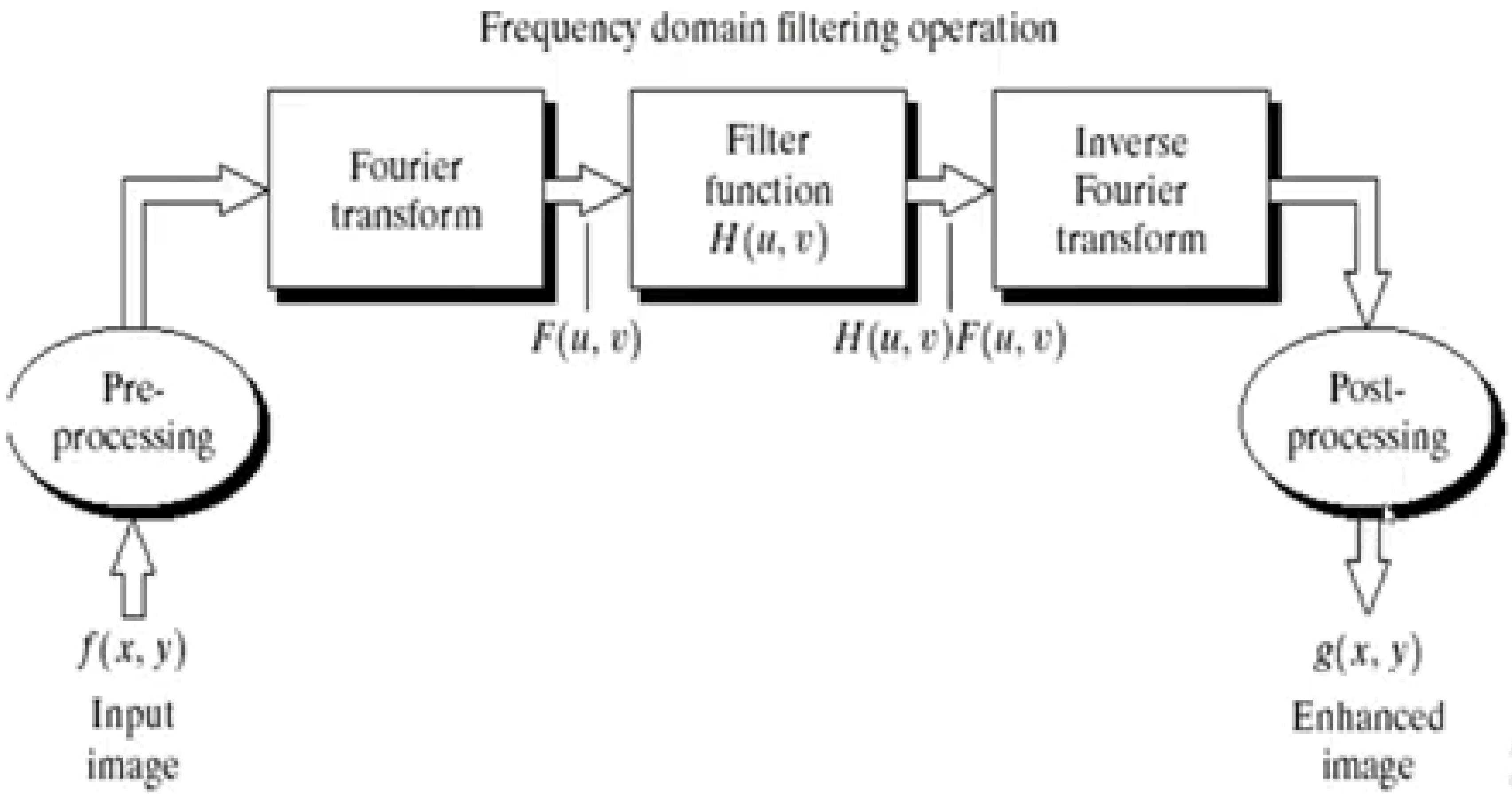
➢ **Basics of filtering in the frequency domain**



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The mathematic equation for DFT is as following :

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)[\cos\left(2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)\right)] - j[\sin\left(2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)\right)]$$

Where the real part is:

$$R(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\cos\left(2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)\right)$$

Where the imaginary part is:

$$I(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\sin\left(2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)\right)$$

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Example : find the DFT for the following image

1	**2**
2	**3**

Solution :

α	0° (0 rad)	30° (π/6)	45° (π/4)	60° (π/3)	90° (π/2)	180° (π)	270° (3π/2)	360° (2π)
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1

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$$M = 2, N = 2, \; f(0,0) = 1, \; f(0,1) = 2, \; f(1,0) = 2, \; f(1,1) = 3$$

$$F(0,0) = \frac{1}{4}\sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)[\cos\left(2\pi\left(\frac{0}{2}+\frac{0}{2}\right)\right)] - j[\sin\left(2\pi\left(\frac{0}{2}+\frac{0}{2}\right)\right)]$$

$$F(0,0) = \frac{1}{4}\sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)[\cos(2\pi(0))] - j[\sin(2\pi(0))]$$

$$F(0,0) = \frac{1}{4}\sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)[\cos(0)] - j[\sin(0)]$$

$$F(0,0) = \frac{1}{4}\sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)[1 - 0]$$

$$F(0,0) = \frac{1}{4}((1+2+2+3)[1]) = \frac{8}{4} = 2$$

$$F(0,1) = \frac{1}{4}\sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)[\cos\left(2\pi\left(\frac{0}{2}+\frac{1y}{2}\right)\right)] - j[\sin\left(2\pi\left(\frac{0}{2}+\frac{1y}{2}\right)\right)]$$

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$$F(0,1) = \frac{1}{4}[((1)[\cos(0)] - j[sin(0)]) + ((2)[\cos(\pi)] - j[sin(\pi)])$$
$$+ ((2)[\cos(0)] - j[sin(0)]) + ((3)[\cos(\pi)] - j[sin(\pi)])]$$

$$F(0,1) = \frac{1}{4}((1 \times 1) + (2 \times -1) + (2 \times 1) + (3 \times -1)) = \frac{-1}{2}$$

$$F(1,0) = \frac{1}{4}\sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)[\cos\left(2\pi\left(\frac{1x}{2} + \frac{0}{2}\right)\right)] - j[sin\left(2\pi\left(\frac{1x}{2} + \frac{0}{2}\right)\right)]$$

$$F(1,0) = \frac{1}{4}[((1)[\cos(0)] - j[sin(0)]) + ((2)[\cos(0)] - j[sin(0)])$$
$$+ ((2)[\cos(\pi)] - j[sin(\pi)]) + ((3)[\cos(\pi)] - j[sin(\pi)])]$$

$$F(1,0) = \frac{1}{4}((1 \times 1) + (2 \times 1) + (2 \times -1) + (3 \times -1)) = -\frac{1}{2}$$

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$$F(1,1) = \frac{1}{4}\sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)[\cos\left(2\pi\left(\frac{1x}{2}+\frac{1y}{2}\right)\right)] - j[\sin\left(2\pi\left(\frac{1x}{2}+\frac{1y}{2}\right)\right)]$$

$$F(1,1) = \frac{1}{4}[((1)[\cos(0)] - j[\sin(0)]) + ((2)[\cos(\pi)] - j[\sin(\pi)])$$
$$+ ((2)[\cos(\pi)] - j[\sin(\pi)]) + ((3)[\cos(2\pi)] - j[\sin(2\pi)])]$$

$$F(1,1) = \frac{1}{4}((1\times 1) + (2\times -1) + (2\times -1) + (3\times 1)) = 0$$

Inverse Discrete Fourier Transform:

The mathematic equation for i-DFT is as following :

$$f(x,y) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(u,v)[\cos\left(2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)\right)] - j[\sin\left(2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)\right)]$$

End of Lecture