University of Al-Hamdaniya
Computer Science Department
Digital image processing


# Histogram Processing and Modification

## Lec-9

By

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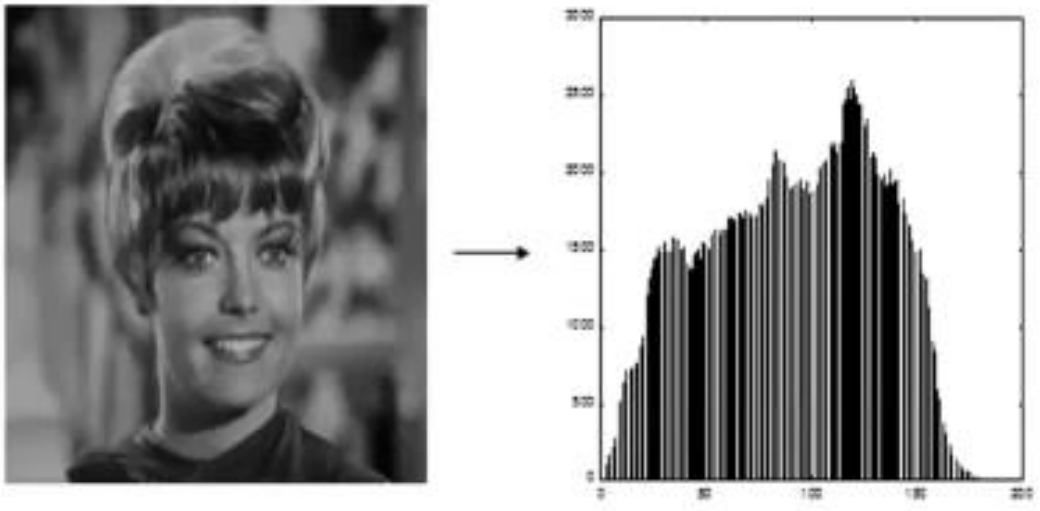
## Histogram Processing

- Histogram of an image represents the relative frequency of occurrence of various gray levels in the image. The histogram of an image is a plot of the gray levels values versus the number of pixels at that value.



Matlab function → imhist(x)

# 2- Histogram processing

The histogram of a digital image with gray levels in the range is discrete function :

$$h(r_k) = n_k$$

where $r_k$ is the $k^{th}$ gray level and $n_k$ is the number of pixels in the image having gray level $r_k$ .

It is common practice to **normalize** a histogram by dividing each of its values by the total number of pixels in the image, denoted by **n**. Thus, a normalized histogram is given by:

$$p(r_k) = \frac{n_k}{n} \qquad \text{For } k = 0, 1, L\text{-}1$$

Loosely speaking, $p(r_k)$ gives an estimate of the probability of occurrence of gray level **rk**.

# Histogram Modification

- **Histogram Modification**
- The gray level histogram of an image is the distribution of the gray level in an image. The histogram can be enhanced or modified by mapping functions, which will stretch, shrink (compress), or slide the histogram.
- Consider **Figure below** shown in four basic gray-level characteristics:

1. **Dark image that the components of the histogram are concentrated on the low (dark) side of the gray scale.**
2. **Light image that the components of the histogram are concentrated on the high (white) side of the gray scale.**
3. **low contrast low contrast has a histogram that will be narrow and will be centered toward the middle of the gray scale**
4. **high contrast high contrast has a histogram that will be distributor though all image of the gray scale**

# Histogram Modification

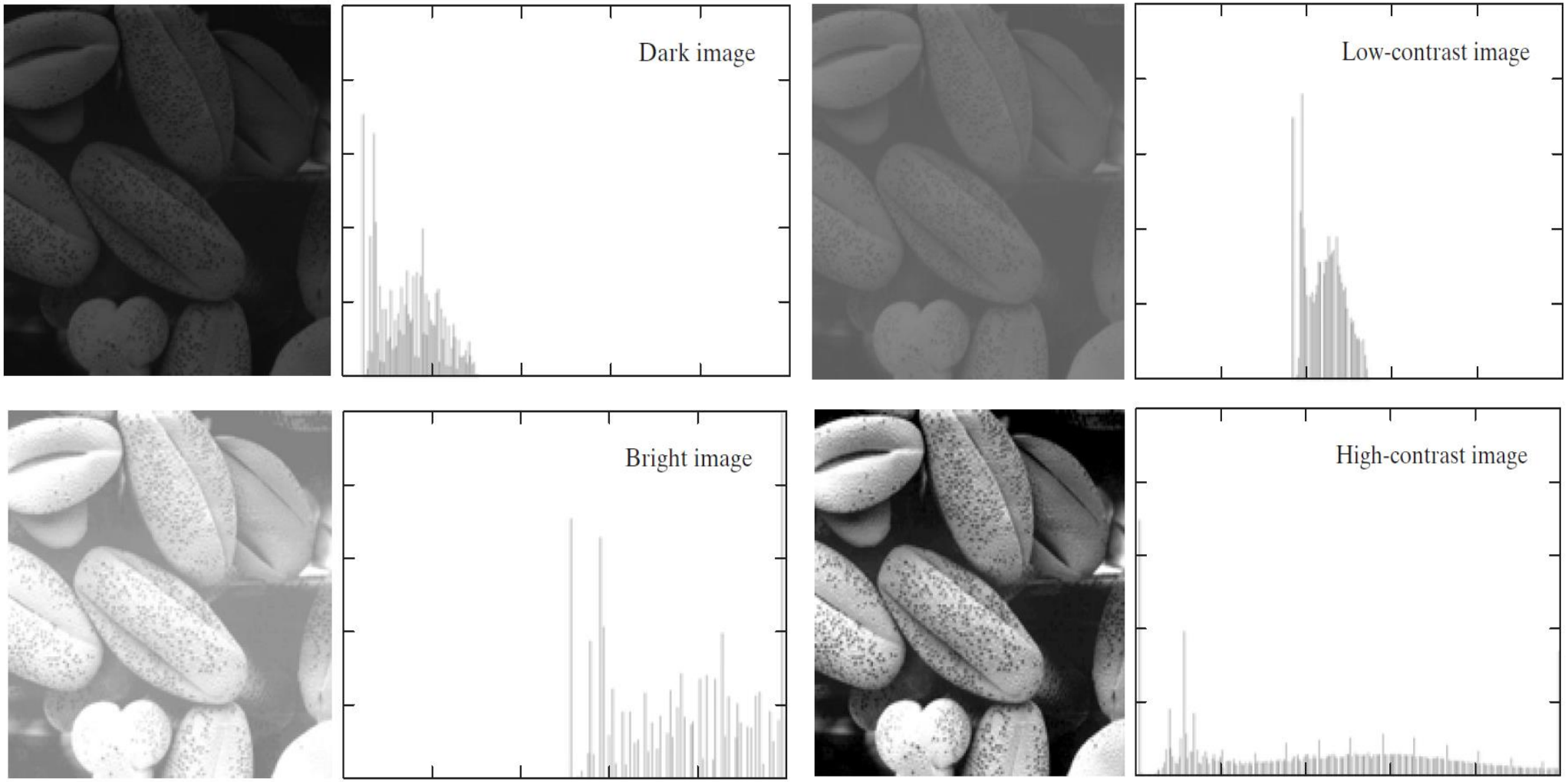


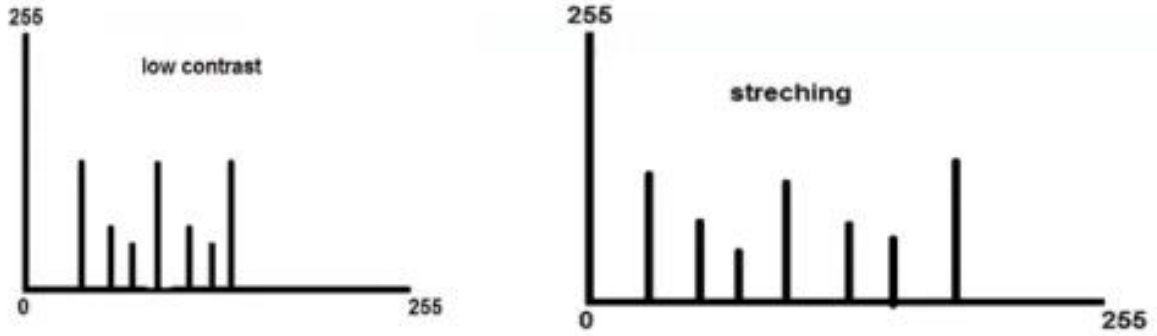

**Figure :** Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

# Histogram Modification

**The histogram changes in three ways:**

1. **Histogram stretching** 2. **Histogram shrink** 3. **Slide of histogram**

- **1. Histogram Stretching**
- **The stretch will be expanding the low contrast of an image as showing in figure below:**




- **And can be executed by using the following equation:**

$$f_{stretch}(x,y) = \left(\frac{f(x,y) - f(x,y)_{min}}{f(x,y)_{max} - f(x,y)_{min}}\right)(Max - Min) + Min$$

- **If we use binary image the (Min, Max)=(0,1)**
- **If we use gray image the (Min, Max)=(0,255)**

# Histogram stretching

**Example:** expand the following image by using stretching:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 50 & 23 \\ 1 & 1 & 2 \end{bmatrix}$$

**Solution**

$f(x,y)_{min} = 1, \quad f(x,y)_{max} = 50 \quad max = 255 \quad min = 0$

$$f_{stretch}(x,y) = \left( \frac{f(x,y) - f(x,y)_{min}}{f(x,y)_{max} - f(x,y)_{min}} \right) (Max - Min) + Min$$

# Histogram stretching

$$f_{stretch}(1,1) = \left(\frac{f(1,1) - 1}{50 - 1}\right)(255 - 0) + 0$$

$$f_{stretch}(1,1) = \left(\frac{1 - 1}{50 - 1}\right)(255 - 0) + 0 = 0$$

$$f_{stretch}(1,2) = \left(\frac{2 - 1}{50 - 1}\right)(255 - 0) + 0 = 5.2$$

$$f_{stretch}(1,3) = \left(\frac{1 - 1}{50 - 1}\right)(255 - 0) + 0 = 0$$

$$f_{stretch}(2,1) = \left(\frac{3 - 1}{50 - 1}\right)(255 - 0) + 0 = 10.4$$

$$f_{stretch}(2,2) = \left(\frac{50 - 1}{50 - 1}\right)(255 - 0) + 0 = 255$$

$$f_{stretch}(2,3) = \left(\frac{23 - 1}{50 - 1}\right)(255 - 0) + 0 = 114.4$$

# Histogram stretching

$$f_{stretch}(3,1) = \left(\frac{1-1}{50-1}\right)(255-0)+0 = 0$$

$$f_{stretch}(3,2) = \left(\frac{1-1}{50-1}\right)(255-0)+0 = 0$$

$$f_{stretch}(3,3) = \left(\frac{2-1}{50-1}\right)(255-0)+0 = 5.2$$

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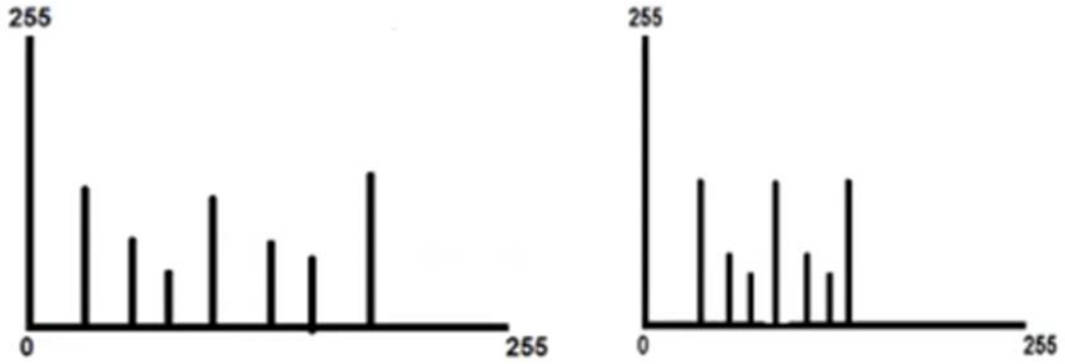
$$f_{stretch}(x,y) = \begin{bmatrix} 0 & 5 & 0 \\ 10 & 255 & 114 \\ 0 & 0 & 5 \end{bmatrix}$$

# Histogram Shrink

## 2. Histogram Shrink

- The opposite of a histogram stretch is a histogram shrink, which will decrease image contrast by compressing the gray levels as showing in figure below:




- **And can be executed by using the following equation:**

$$Shrink(f(x,y)) = \left[\frac{shrink_{\max} - shrink_{\min}}{f(x,y)_{\max} - f(x,y)_{\min}}\right]\left[f(x,y) - f(x,y)_{\min}\right] + shrink_{\min}$$

- **(*Shrink*$_{max}$, *Shrink*$_{min}$)** = correspond to the maximum and minimum desired in the compressed histogram.

# Histogram Shrink

**Example: reduce the histogram for following sub image :**

$$f(x,y) = \begin{bmatrix} 70 & 120 & 80 \\ 200 & 90 & 60 \\ 100 & 150 & 10 \end{bmatrix}$$

**Where $Shrink_{max}$ = 100 and $Shrink_{min}$ = 20**

$$Shrink\ f(1,1) = \left(\frac{100-20}{200-10}\right)(70-10)+20 = 45$$

$$Shrink\ f(1,2) = \left(\frac{100-20}{200-10}\right)(120-10)+20 = 66$$

$$Shrink\ f(1,3) = \left(\frac{100-20}{200-10}\right)(80-10)+20 = 49$$

# Histogram Shrink

$$\boldsymbol{Shrink\ f(2,1)} = \left(\frac{100-20}{200-10}\right)(200-10)+20 = \mathbf{100}$$

$$\boldsymbol{Shrink\ f(2,2)} = \left(\frac{100-20}{200-10}\right)(90-10)+20 = \mathbf{54}$$

$$\boldsymbol{Shrink\ f(2,3)} = \left(\frac{100-20}{200-10}\right)(60-10)+20 = \mathbf{41}$$

$$\boldsymbol{Shrink\ f(3,1)} = \left(\frac{100-20}{200-10}\right)(100-10)+20 = \mathbf{58}$$

$$\boldsymbol{Shrink\ f(3,2)} = \left(\frac{100-20}{200-10}\right)(150-10)+20 = \mathbf{79}$$

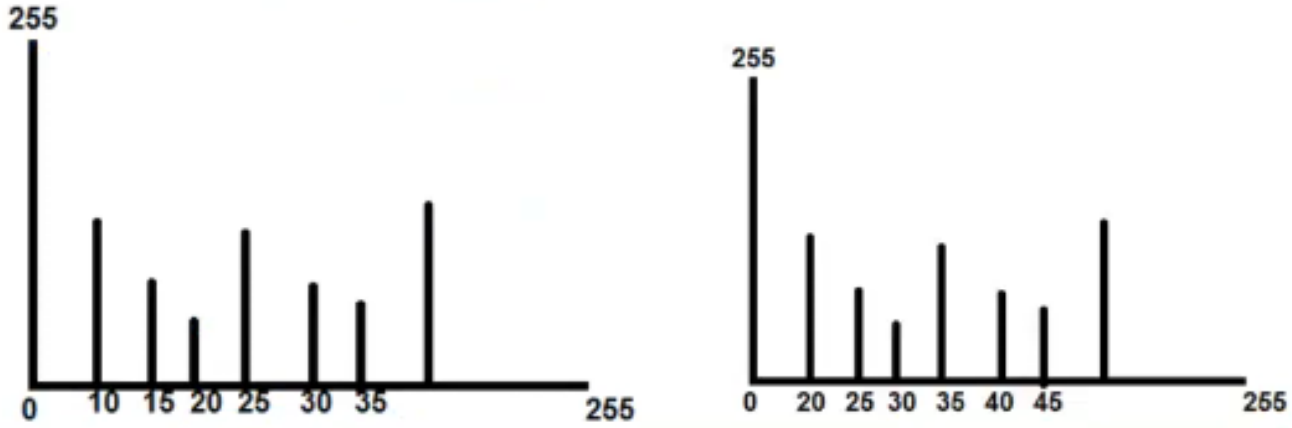
$$\boldsymbol{Shrink\ f(3,3)} = \left(\frac{100-20}{200-10}\right)(10-10)+20 = \mathbf{20}$$

$$\boldsymbol{Shrink\ f(x,y)} = \begin{bmatrix} \mathbf{45} & \mathbf{66} & \mathbf{49} \\ \mathbf{100} & \mathbf{54} & \mathbf{41} \\ \mathbf{58} & \mathbf{79} & \mathbf{20} \end{bmatrix}$$

# Histogram Slide

## 3. Histogram slide

- The histogram slide techniques can be used to make an image either darker or lighter but retain the relationship between gray-level values. This can be a accomplished by simply adding or subtracting a fixed number for all the gray-level values, as showing in figure below:




**And can be executed by using the following equation:**

$$f_{slide}(x,y) = f(x,y) + offset$$

# Histogram Slide

**Example:** shift the following image by (10)

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 50 & 23 \\ 1 & 1 & 2 \end{bmatrix}$$

$$f_{slide}(x,y) = f(x,y) + offset$$

$$f_{slide}(0,0) = f(0,0) + 10$$

$$f_{slide}(0,0) = 1 + 10 = 11$$

$$f_{slide}(0,1) = 2 + 10 = 12$$

$$f_{slide}(0,2) = 1 + 10 = 11$$

$$f_{slide}(1,0) = 3 + 10 = 13$$

$$f_{slide}(1,1) = 50 + 10 = 60$$

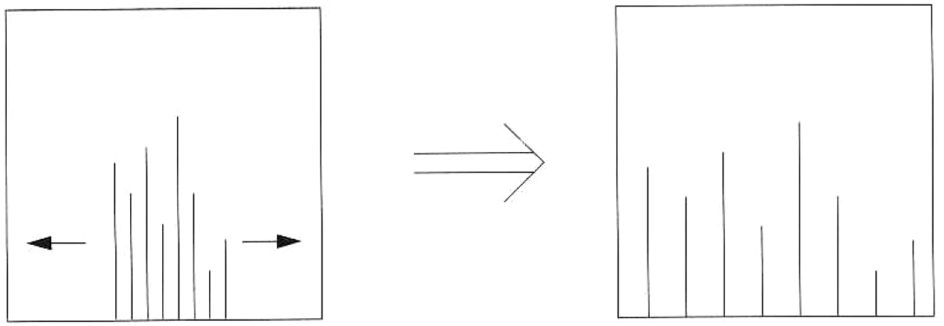
$$f_{slide}(1,2) = 23 + 10 = 33$$

$$f_{slide}(2,0) = 1 + 10 = 11$$

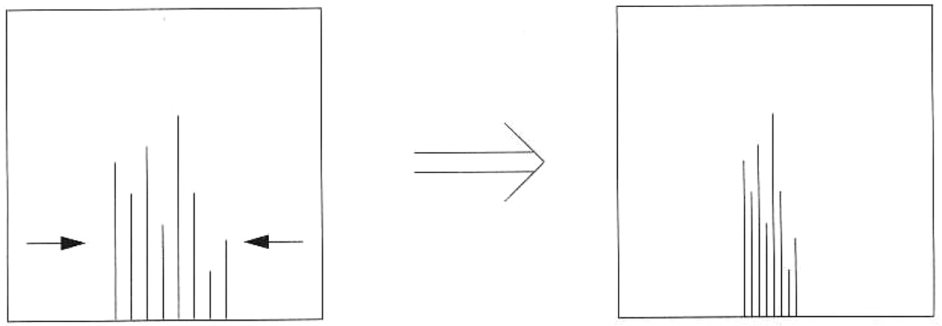
$$f_{slide}(2,1) = 1 + 10 = 11$$

$$f_{slide}(2,2) = 2 + 10 = 12$$

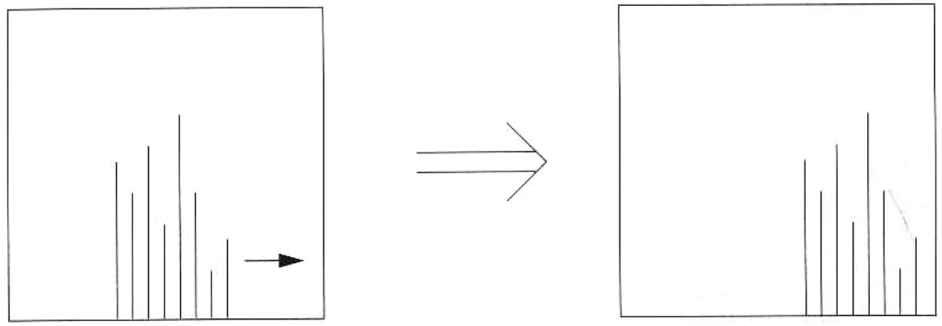
$$f_{slide}(x,y) = \begin{bmatrix} 11 & 12 & 11 \\ 13 & 60 & 33 \\ 11 & 11 & 12 \end{bmatrix}$$



a. Histogram stretch.



b. Histogram shrink.



c. Histogram slide.

**Summary of Histogram Modifications.**

# End of Lecture