Al-Hamdaniya University

College of Education

Computer Science

Stage: 2nd

Grammar in Automata Theory

Grammar in automata theory refers to a set of rules that define the structure of strings in a language. These rules are used to generate all possible strings in a particular language, thereby defining the language itself.

What is Linear Grammar?

A linear grammar is a type of formal grammar where each production rule has at most one non-terminal symbol on the right-hand side of the production.

Linear grammar is significant because it describes a subset of regular languages and plays an important role in the design and analysis of finite automata.

Examples of Linear Grammar

Let us see the idea of linear grammar through examples. We will see both the left and right linear grammars one by one for a better understanding.

Example of Left-Linear Grammar

Consider the following left-linear grammar production rules

$$S \rightarrow Ab$$

$$A \rightarrow Bb$$

$$B \rightarrow a$$

- The start symbol S produces Ab.
- The non-terminal A produces Bb.
- Finally, B produces a.

Example of Right-Linear Grammar

Consider the following right-linear grammar production rules –

$$S \rightarrow aA$$

$$A \rightarrow bB$$

$$B \rightarrow c$$

- The start symbol S produces aA.
- The non-terminal A produces bB.
- Finally, B produces c.

Chomsky Normal Form (CNF)

A context free grammar (CFG) is in Chomsky Normal Form (CNF) if all production rules satisfy one of the following conditions:

- If the start symbol S occurs on some right side, create a new start symbol S' and a new production S'→ S.
- A non-terminal generating a terminal (e.g.; X->x)
- A non-terminal generating two non-terminals (e.g.; X->YZ)
- Eliminate Null.

Convert A context free grammar (CFG) into Chomsky Normal Form

Problem1

Convert the following CFG into CNF

S ⟶ aSa | bSb | a | b | aa | bb

solution

S ⟶ ASA | BSB | a | b | AA | BB

A ⟶ a

B ⟶ b

R1 ⟶ SA

R2 ⟶ SB

S ⟶ AR1 | BR2 | a | b | AA | BB |

Problem 2

S ⟶ bA | aB

A ⟶ bAA | aS | a

B ⟶ aBB | bS | b

Solution:

W ⟶ b Z ⟶ a

S ⟶ WA | ZB

A ⟶ WAA | ZS | a R ⟶ AA

A ⟶ WR | ZS | a

B ⟶ ZBB | WS | b X ⟶ BB

B ⟶ ZX | WS | b

Problem 3

$S \longrightarrow Xa$

$X \longrightarrow aX \mid bX \mid \varepsilon$

Solution

$S \longrightarrow Xa \mid a$

$X \longrightarrow aX \mid bX \mid a \mid b$

Is it CNF ?

Problem 4

$S \longrightarrow AAS \mid aB$

$A \longrightarrow B \mid S$

$B \longrightarrow b \mid \varepsilon$

Solution:

$B \longrightarrow \varepsilon$, $A \longrightarrow \varepsilon$

$S \longrightarrow AAS \mid aB \mid a \mid AS \mid AS \mid S$

$A \longrightarrow B \mid S$

$B \longrightarrow b$

$H \longrightarrow AA$

$T \longrightarrow a$

$S \longrightarrow HS \mid TB \mid T \mid AS \mid AS \mid S$