Al-Hamdaniya University

College of Education

Computer Science

Stage: 2nd

- **Theory of Computation** is divided into three major branches:

1- **Automata theory:** Automata Theory deals with definitions and properties of different types of "computation models". Examples of such models are:

- Finite Automata: These are used in text processing, compilers, and hardware design.
- Context-Free Grammars: These are used to define programming languagesand in Artificial Intelligence.
- Turing Machines: These form a simple abstract model of a "real" computer, such as your PC at home.

2- **Computability theory:** Computability theory deals primarily with the question of the extent to which a problem is solvable on a computer. In otherwords, classify problems as being solvable or unsolvable.

3- **Complexity theory:** Complexity theory considers not only whether a problem can be solved at all on a computer, but also how efficiently the problem can be solved. Two major aspects are considered:

- Time complexity: and how many steps does it take to perform a computation.
- Space complexity: how much memory is required to perform that computation.

➢ **Some Applications of Computation Theory:**

1. Design and Analysis of Algorithms.
2. Computational Complexity.
3. Logic in Computer Science.
4. Compiler.
5. Cryptography.
6. Randomness in Computation.
7. Quantum Computation

Sets

A set is a collection of “objects” called the elements or members of the set.

Common forms of describing sets are:

- List all elements, e.g. {a, b, c, d}.
- Form new sets by combining sets through operators.

Examples in Sets Representation:

- C = { a, b, c, d, e, f }   finite set
- S = { 2, 4, 6, 8, …}   infinite set
- S = { j : j > 0, and j = 2k for k > 0 }
- S = { j : j is nonnegative and even }

Terminology and Notation:

- To indicate that x is a member of set **S**, we write **x**∈ **S**.
- To denote the empty set (the set with no members) as **{}** or **∅**.

- If every element of set **A** is also an element in set **B**, we say that **A** is a subset of **B**, and write **A**$\subseteq$ **B** or **B**$\supseteq$**A**.
- If **A** is not a part of **B**, if at least one of the elements of **A** does not belong to **B** then we say that **A** is not a subset of **B**, and write **A**$\nsubseteq$ **B** or **B**$\nsupseteq$**A**.

Basic Operations on Sets:

- **Complement:** Á or $\bar{A}$ or A^c
  $\bar{A} = \{x: x \notin A, x \in U\}$
  Contain all elements in universal set which are not in **A**.
- **Union:** consist of all elements in either **A** or **B**
  $A \cup B = \{ x:x \in A \text{ or } x \in B\}$

- **Intersection:** consist of all elements in both A or B $A \cap B = \{ x:x \in A \text{ and } x \in B\}$

- **Difference (/):** consist of all elements in A but not in B $A / B = \{ x:x \in A \text{ but } x \notin B\}$

Properties of Sets:

Let **A**, **B**, and **C** be subsets of the universal set **U**.

- **Distributive properties**
  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- **Idempotent properties**
  $A \cap A = A$. $A \cup A = A$.
- **Double Complement property**
  $(A^{\sim})^{\sim} = A$.

- **De Morgan's laws**
  $(A \cup B)^{\sim} = A^{\sim} \cap B^{\sim}$
  $(A \cap B)^{\sim} = A^{\sim} \cup B^{\sim}$
- **Commutative properties**
  $A \cap B = B \cap A$.
  $A \cup B = B \cup A$.
- **Associative laws**
  $A \cap (B \cap C) = (A \cap B) \cap C$
  $A \cup (B \cup C) = (A \cup B) \cup C$
- **Identity properties**
  $A \cup \emptyset = A$.
  $A \cap U = A$.
- **Complement properties**
  $A \cup A^{\sim} = U$. $A \cap A^{\sim} = \emptyset$.

Language

Language is the set of strings of terminal symbols derivable from alphabet.

Symbol: are an entity or individual objects, which can be any letter alphabet, or any picture.

Ex: 1, a, b, #

Alphabets are a finite set of symbols. It denoted by **(∑)**.

Ex:
∑ = {a, b}
∑ = {A, B, C, D}
∑ = {0, 1, 2}
∑ = {#, β, △ }

String: It is a finite collection of symbol from the alphabet. The string denoted by w.

Ex: If ∑ = {a, b} various string that can be generated from
∑ are {ab, aa, aaa, bb, bbb, ba, aba, …..}

- ❖ A string with no symbol is known as an empty string. It is represented by epsilon (ϵ) or Lambda(λ) or null(∧).
- ❖ The number of symbols in a string w is called the length of string. It is denoted by |w|.

Ex:
w=010
|w|= 3
|00100|=5
|ab|=2
||= 0

Types of Languages:

1- ***Natural Languages:*** They are languages that spoken by humans e.g.: English, Arabic and France. It has alphabet: ∑={**a**, **b**, **c**, **…. z**}. From these alphabetic we make sentences that belong to the language.

2- ***Programming Language:*** (e.g.: Python, Java, C++) it has alphabetic: ∑= {a, b, c, z, A, B, C, .. , Z , ?, /, -, \}. From these alphabetic we make sentences that belong to programming language.

Ex:

Alphabetic: ∑= {0, 1}.

Sentences: 0000001, 1010101.

Alphabetic: ∑= {a, b}. Sentences: ababaabb, bababbabb

Example:

Let ∑ = {x} be set of alphabet of one letter x. we can write this in form: L_1 = {x, xx, xxx, …}

or write this in an alternate form:

$L_1 = \{x^n$ for n = 1, 2, 3, ...}

Let a = xxx and b = xxxxx Then ab = xxxxxxxx = x^8

ba = xxxxxxxx = x^8

Example:

L_2 = { x, xxx, xxxxx, ... }

= { x^{odd}}

= { x^{2n+1} for n = 0, 1, 2, 3, ... }

Palindrome

Let us define a new language called **Palindrome** over the alphabet

∑= {a, b}

PALINDROME = { **Λ,** and all strings x such that reverse(x) = x } If we begin listing the elements in PALINDROME we find:

PALINDROME = { **Λ,** a, b, aa, bb, aaa, aba, bab, bbb, aaaa, abba, ... }

Kleene Closuer

They are two repetition marks, also called Closuer or Kleene Star.

***** : Repeat (0 – n) times.

+ : Repeat (1 – n) times.

Example:

If $\sum = \{x\}$, then

$\sum^{*} = L_3 = \{\wedge, x, xx, xxx,\ldots\}$

$\sum^{+} = L_3 = \{x, xx, xxx, \ldots\}$

Example:

If $\sum = \{0, 1\}$, then

$\sum^{*} = L_4 = \{\wedge, 0, 11, 001, 11010, \ldots\}$

$\sum^{+} = L_4 = \{0, 01, 110, 101, \ldots\}$

Example:

If $\sum = \{aa, b\}$, then

$\sum^{*} = L_5 = \{ \wedge, aab, baa, baab, aabb, \ldots\}$

$\sum^{+} = L_5 = \{ aaaa, b, baaaa, bb, \ldots\}$

Note: The word (ab) is not acceptable in this language because (aa) is a single letter that cannot be divided into parts.

<u>Example:</u>

If $\sum = \{ \}$, then

$\sum^{*} = L_4 = \{\wedge\}$

$\sum^{+} = L_4 = \emptyset$ or { }