الفصل السادس

الدوال المتسامية

Transcendental Functions

(1-6) المقدمة

الدوال المتسامية:- وهي كل دالة ليست جبرية والدوال المتسامية البسيطة تشمل الدوال اللوغارتميه والدوال الآسية والدوال المثلثيه والدوال المثلثية العكسية والى أخرى من الدوال.

(2-6)الدوال اللوغارتمية Logarithmic Functions

تقسم الدوال اللوغارتمية إلى قسمين:-

- اللوغارتم الطبيعي Natural log:-

$$\log_e(x) = \ln(x)$$
$$e \cong 2.7$$

- اللوغارتم الطبيعي Normal log:-

$$\log^x(x)$$
$$a :- positive.real.nomber$$

****Natural Lagarithm function****

$$\log_e(x) = \ln(x)$$
$$e \cong 2.7$$

Properties of the f(x) = ln(x)

$$1/\ D(\ln(x)) = (0,\infty)$$
$$2/\ R(\ln(x)) = (-\infty,\infty)$$
$$3/\ contiuous, increa\sin g$$
$$4/\ the.graph.concav.dorwn$$

Properties of the logarithm

$$1/\ \ln(1) = 0$$
$$2/\ \ln(a^b) = b\ln(a)$$
$$3/\ \ln(ab) = \ln(a) + \ln(b)$$
$$4/\ \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$
$$5/\ \ln(\frac{1}{a}) = -\ln(a)$$
$$6/\ \ln(e(x)) = x$$
$$7/\ e^{\ln(x)} = x$$

قواعد الحل للدالة اللوغارتمية الطبيعة

$$1/\ \ln(x) = \int_1^x \frac{1}{t}dt \to \frac{d(\ln(x))}{dx} = \frac{1}{x}$$
$$2/\ \frac{d\ln}{dx}\left[f(x)\right] = \frac{1}{f(x)}.f'(x)$$
$$3/\ \int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$4/\int \frac{dx}{x^2-a^2}=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+c$$

$$5/\int \frac{dx}{(x+a)(x+b)}=\frac{1}{b-a}\ln\left|\frac{x+a}{x+b}\right|+c$$

أمثلة عن اشتقاق الدالة اللوغارتمية الطبيعية

$$EX1: y=\frac{\ln(x)}{x^2+1}\rightarrow find\,(y')$$

$sol:-$

$$y'=\frac{(x^2+1)(\frac{1}{x})-(2x)(\ln(x))}{(x^2+1)^2}=\frac{(x^2+1)(\frac{1}{x})-(2x)(\ln(x))}{(x^2+1)^2}$$

$$EX2: y=\ln(x\sqrt{1-x^2})\rightarrow find\,(y')$$

$sol:-$

$$y=\ln(x)+\ln\sqrt{1-x^2}\rightarrow \ln(1-x^2)^{\frac{1}{2}}=\frac{1}{2}\ln(1-x^2)$$

$$y'=\frac{1}{x}+\frac{1}{2}\left(\frac{-2x}{1-x^2}\right)=\frac{1}{x}-\frac{x}{1-x^2}=\frac{1-x^2-x^2}{x(1-x^2)}=\frac{1-2x^2}{x(1-x^2)}$$

$$EX3: y=\ln|2x-1|\rightarrow find\,(y')$$

$sol:-$

$y=2x-1$

$$f'(x)=\frac{1}{2x-1}\times 2=\frac{2}{2x-1}$$

$$EX4: y=x^{\ln(x)}\rightarrow find\,(y')$$

$sol:-$

$\ln y=\ln(x)^{\ln(x)}$

$\ln y=(\ln(x))(\ln(x))$

$\ln(y)=\left[\left(\ln(x)^2\right)\right]$

$$\frac{1}{y}*y'=2(\ln(x))(\frac{1}{x})$$

$$y'=y(\frac{2\ln(x)}{x})$$

$$x^{\ln(x)}=\left(\frac{2\ln(x)}{x}\right)$$

$EX5: y = x^2 \ln(x\cos(x)) \rightarrow find\,(y')$

$sol:-$

$$y' = x^2 . \frac{1}{x\cos(x)}[x - \sin(x) + \cos(x)] + \ln(x\cos(x))2x$$

$EX5: y = \ln(\sin(x)) \rightarrow find\,(y')$

$sol:-$

$$y' = \frac{1}{\sin(x)}.\cos(x) = \frac{\cos(x)}{\sin(x)} = \cot(x)$$

أمثلة عن تكامل الدالة اللوغارتمية الطبيعية

$EX1: find : y = \int \cot(x)$

$sol:-$

$$y = \int \frac{\cos(x)}{\sin(x)}dx = \ln|\sin(x)| + c$$

$EX2: find : y = \int \tan(x)dx$

$sol:-$

$$y = \int \frac{\sin(x)}{\cos(x)}dx = -\ln|\cos(x)| + c$$

$EX3: find : y = \int \frac{3x^2+1}{x^3+x}dx$

$sol:-$

$$y = \ln|x^3 + x| + c$$

$EX4: find : y = \int \frac{dx}{9x^2-25}$

$sol:-$

$$y = \int \frac{dx}{(3x)^2 - (5)^2} = \frac{1}{(2)(3)(5)} \ln\left|\frac{3x-5}{3x+5}\right| + c$$

$EX5: find : y = \int \frac{dx}{4x^2+4x-3}dx$

$sol:-$

$$y = \int \frac{dx}{(2x+3)(2x-1)} = \int \frac{dx}{(2)(2)\left(x+\frac{3}{2}\right)(x-\frac{1}{2})} = \frac{1}{4}\int \frac{dx}{(x+3/2)(x-1/2)} =$$

$$= \frac{1}{4}.\frac{1}{3/2+1/2}\ln\left|\frac{x-1/2}{x+3/2}\right| + c = \frac{1}{8}\ln\left|\frac{2x-1}{2x+3}\right| + c$$

****Normal Lagarithm function****

$$f(x) = \log_a(u)$$
$$a :- positive.real.nomber$$
$$u :- any.function$$

Properties of the f(x) = log$_a$(x)

$$1/\log_a(u_1 * u_2) = \log_a u_1 + \log_a u_2$$
$$2/\log_a(\frac{u_1}{u_2}) = \log_a u_1 - \log_a u_2$$
$$3/\log_a u^x = x\log_a u$$
$$4/\log_a u = \frac{\ln u}{\ln a}$$
$$5/\log_a u^x = x$$
$$6/a^{\log_a x} = x$$
$$7/\frac{d}{dx}\log_a u = \frac{1}{u.\ln(a)}.\frac{du}{dx}$$ حفظ

وبشكل عام لا يوجد قانون ثابت في الدوال اللوغارتمية لا يجاد التكامل.

أمثلة عن اشتقاق الدالة اللوغارتمية الاعتيادية

$$EX1: y = \log_7 x^3 \rightarrow find(y')$$
$$sol :-$$
$$y' = \frac{1}{x^3.\ln(7)}.3x^2 = \frac{3x^2}{x^3\ln(7)} = \frac{3}{x\ln(7)}$$

$$EX2: y = \log_2 \frac{\sin(x)}{x^2} \rightarrow find(y')$$
$$sol :-$$
$$y = \log_2 \sin(x) - \log_2 x^2$$
$$\frac{dy}{dx} = \frac{1}{\sin(x)\ln(2)}.\cos(x) - \frac{1}{x^2\ln(2)}.2x = \frac{\cos(x)}{\sin(x)\ln(2)} - \frac{2}{x\ln(2)} = \frac{\cot(x)}{\ln(2)} - \frac{2}{x\ln(2)}$$

$$EX3: y = \log_{100}(\ln(x))^4 \rightarrow find(y')$$
$$sol :-$$
$$y = 4\log_{100}(\ln(x))$$
$$\frac{dy}{dx} = 4.\frac{1}{\ln(x)\ln(100)}.\frac{1}{x} = \frac{4}{x\ln(100)\ln(x)}$$

$EX\,4: y = \log_{2.7} x^2 \to find\,(y')$

$sol:-$

$e \cong 2.7 = \log_{e} x^2 = \ln(x^2)$

$$\frac{dy}{dx} = \frac{1}{x^2}.2x = \frac{2}{x}$$

$$\frac{dy}{dx} = \frac{1}{x^2\ln(2.7)}.2x = \frac{2}{x\ln(e)} = \frac{2}{x}$$

$EX\,5: y = \log_{33}\frac{(x^2+3)^2}{\ln(x)} \to find\,(y')$

$sol:-$

$y = \log_{33}(x^2+3)^2 - \log_{33}(\ln x^2)$

$y = 2\log_{33}(x^2+3) - \log_{33}(\ln(x^2))$

$$\frac{dy}{dx} = \frac{2}{(x^2+3)\ln(33)}.2x - \frac{1}{\ln(x^2).\ln 33}.\frac{1}{x^2}.2x$$

$$= \frac{4x}{(x^2-3)\ln 33} - \frac{2}{x\ln(x^2)\ln 33}$$

(3-6)الدوال الآسية Exponential Functions

تقسم الدوال الآسية:-

- f(x) a^u -: Exponential Functions
- f(x)= e^u -:Exponential Functions

$1/\ y = e^x \to inverse \to \ln(x)$

$2/\ D(e^x) = (-\infty,\infty) \to e^x$

$3/\ R(e^x) = (0,\infty) \to e^x$

$4/\ contnous, increasing, concave up$

$5/\ \ln(e^x) = x; e^{\ln(x)} = x$

$6/\ e^a.e^b = e^{a+b}$

$7/\ e^{-a} = \frac{1}{e^a}$

$8/\ \frac{e^a}{e^b} = e^{a-b}$

$9/\ e^0 = 1$

$10/\ (e^a)^b = e^{ab}$

$11/\ \lim_{x\to\infty} e^x = \infty$

$12/\ \lim_{x\to-\infty} e^x = 0$

$13/\ \frac{de^u}{dx} = e^u.\frac{du}{dx}, \to$ حفظ

$14/\int e^{u}du=e^{u}+c\rightarrow$ حفظ

أمثلة عن اشتقاق الدالة الآسية $f(x) = e^u$

$$EX1: y=e^{\sin(x)}\rightarrow find(y')$$
$sol:-$
$$y'=e^{\sin(x)}.\cos(x)$$

$$EX2: y=e^{xy}\rightarrow find(y')$$
$sol:-$
$$2y.\frac{dy}{dx}=e^{xy}(x.\frac{dy}{dx}+y.1)$$
$$2y.\frac{dy}{dx}=xe^{xy}.\frac{dy}{dx}+ye^{xy}$$
$$(2y-xe^{xy})\frac{dy}{dx}=ye^{xy}$$
$$\frac{dy}{dx}=\frac{ye^{xy}}{2y-xe^{xy}}$$

$$EX3: y=e^{e^{x}}\rightarrow find(y')$$
$sol:-$
$$y'=e^{e^{x}}(e^{x}.1)=e^{x}e^{e^{x}}$$

$$EX4: y=\ln(\sqrt{e^{x}})\rightarrow find(y')$$
$sol:-$
$$y'=\frac{1}{\sqrt{e^{x}}}.\frac{e^{x}}{2\sqrt{e^{x}}}=\frac{e^{x}}{2e^{x}}=\frac{1}{2}$$

$$EX5: y=\ln(e^{x}+e^{-x})\rightarrow find(y')$$
$sol:-$
$$y'=\frac{1}{e^{x}+e^{-x}}.e^{x}-e^{-x}=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

$$EX6: y=e^{\ln(x^{2}+1)^{2}}\rightarrow find(y')$$
$sol:-$
$$y=e^{\ln(x^{2}+1)^{2}}=(x^{2}+1)^{2}$$
$$y'=2(x^{2}+1)(2x)=4x(x^{2}+1)$$

ملاحظة :- لإيجاد التكامل لأي دالة يجب توفير المشتقة للدالة ومن ثم إجراء التكامل حسب نوع الدالة المعطاة.

أمثلة عن تكامل الدالة الآسية $f(x) = e^u$

$$EX1:\ find:\ y=\int \sec^2(x)e^{\tan(x)}dx$$

$$sol:-$$

$$e^u = e^{\tan(x)};du=\sec^2(x)dx$$

$$\int e^{\tan(x)}\sec^2(x)dx = e^{\tan(x)}+c$$

$$EX2:\ find:\ y=\int e^{3x}dx$$

$$sol:-$$

$$e^u = e^{3x};du=3dx$$

$$\int e^{3x}dx=\frac{1}{3}e^{3x}+c$$

$$EX3:\ find:\ y=\int x\csc(x^2)\cot(x^2)e^{\csc(x^2)}dx$$

$$sol:-$$

$$e^u = e^{\csc(x^2)};du=\csc(x^2).\cot(x^2).2xdx$$

$$y=\frac{1}{2}e^{\csc(x^2)}+c$$

$$EX4:\ find:\ y=\int \left(e^x+e^{-x}\right)dx$$

$$sol:-$$

$$y=\int e^x dx+\int e^{-x}dx$$

$$e^u = e^x, e^{-x} \to dx_1=1, dx_2=-1$$

$$y=e^x-e^{-x}+c$$

$$EX5:\ find:\ y=\int \frac{(e^x-1)^2}{e^x}dx$$

$$sol:-$$

$$y=\int \frac{e^{2x}-2e^x+1}{e^x}dx=\int (e^{2x}-2e^x+1)e^{-x}dx=\int (e^x-2+e^{-x})dx$$

$$y=\int e^x dx-2\int dx+\int e^{-x}dx$$

$$y=e^x-2x-e^{-x}+c$$

$$EX6:\ find:\ y=\int e^{3x+1}dx$$

$$sol:-$$

$$e^u = e^{3x+1}, dx=3$$

$$y=\frac{1}{3}e^{3x+1}+c \to \frac{e^{3x+1}}{3}+c$$

• Exponential Functions :- $f(x) = a^u$

U = is function , a:- positive real number

Properties of the $f(x) = a^u$

$$1/\ a^{x_1}.a^{x_2} = a^{x_1+x_2}$$

$$2/\ \frac{a^{x_1}}{a^{x_2}} = a^{x_1-x_2}$$

$$3/(a^{x_1})^{x_2} = a^{x.x_{21}}$$

$$4/\ a^{-x_1} = \frac{1}{a^{x_1}}$$

$$5/\ a^x = e^{x\ln(a)}$$

$$6/\ \frac{da^u}{dx} = a^u.\ln(a).\frac{du}{dx} \rightarrow \text{حفظ}$$

$$7/\int a^u du = \frac{a^u}{\ln(a)} + c \rightarrow \text{حفظ}$$

أمثلة عن اشتقاق الدالة الآسية $f(x) = a^u$

$$EX1: y = 7.5^{\cos(x^3)} \rightarrow find\,(y')$$
$$sol:-$$
$$y' = 7.5^{\cos(x^3)}.\ln 7.5.(-\sin x^3)(3x^2)$$

$$EX2: y = 25^{e^x} \rightarrow find\,(y')$$
$$sol:-$$
$$y' = 25^{e^x}.\ln 25.e^x.1$$

$$EX3: y = 20^{x^3} \rightarrow find\,(y')$$
$$sol:-$$
$$y' = 20^{x^3}.\ln 20.(3x^2)$$

$$EX4: y = 7^{x^3}\ln(x^2+5) \rightarrow find\,(y')$$
$$sol:-$$
$$y' = 7^{x^3}.\frac{1}{x^2+5}.2x + \ln(x^2+5).7^{x^3}.\ln(7).3x^2$$
$$y' = 7^{x^3}.\frac{2x}{x^2+5} + 3x^2.7^{x^3}\ln(7).\ln(x^2+5)$$

$EX\,5: y = \log_7 (x^3 + 7x) \to find\,(y')$

$sol:-$

$$y' = \frac{1}{(x^3+7x)\ln(7)}.3x^2 + 7 = \frac{3x^2+7}{(x^3+7x)\ln 7}$$

ملاحظة:- إذا كانت الدالة الآسية باس وأساس لنفس المتغير نأخذ (ln) للطرفين ومن ثم نشتق ضمنيا.

$EX\,6: y = x^x \to find\,(y')$

$sol:-$

$$\ln(y) = \ln(x^x)$$
$$\ln(y) = x\ln(x)$$
$$\frac{1}{y}.y' = \left[x.\frac{1}{x} + \ln(x).1\right]$$
$$\frac{y'}{y} = [1+\ln(x)]$$
$$\frac{y'}{y}.y = [1+\ln(x)].y$$
$$y' = [1+\ln(x)]x^x$$

أمثلة عن تكامل الدالة الآسية $f(x) = a^u$

$EX\,1: find : y = \int 2x.9^{x^2} dx$

$sol:-$

$a^u = 9^{x^2}, du = 2xdx$

$$\int 9^{x^2} 2xdx = \frac{9^{x^2}}{\ln(9)} + c$$

$EX\,2: find : y = \int 36^{\cot(x)} \csc^2(x)dx$

$sol:-$

$a^u = 36^{\cot(x)}, du = \csc^2 dx$

$$\int 36^{\cot(x)} \csc^2(x)dx = \frac{36^{\cot(x)}}{\ln(36)} + c$$

$EX\,3: find : y = \int e^x.10.3^{e^x} dx$

$sol:-$

$a^u = 10.3^{e^x}, du = e^x dx$

$$\int e^x.10.3^{e^x} dx = \frac{10.3^{e^x}}{\ln(10.3)} + c$$

$$EX\,4:\ find\ :y=\int (\sin(x).\frac{1}{x}+\ln(x)\cos(x)).3.7^{\sin(x)\ln(x)}dx$$

$sol:-$

$$a^{u}=3.7^{\sin(x)\ln(x)},du=(\frac{\sin(x)}{x}+\ln(x)\cos(x)dx$$

$$y=\frac{3.7^{\sin(x)\ln(x)}}{\ln(3.7)}+c$$

$$EX\,5:\ find\ :y=\int_{-1}^{1}(2^{x+1})dx$$

$sol:-$

$$y=\int_{-1}^{1}2^{x}.2^{1}dx=2\int_{-1}^{1}2^{x}dx$$

$$a^{u}=2^{x},du=dx$$

$$=2\left[\frac{2^{x}}{\ln(2)}\right]_{-1}^{1}=\frac{2(2)^{1}}{\ln(2)}-\frac{2(2)^{-1}}{\ln(2)}$$

$$=\frac{4}{\ln(2)}-\frac{2}{2\ln(2)}=\frac{4}{\ln(2)}-\frac{1}{\ln(2)}=\frac{3}{\ln(2)}$$

(4-6) الدوال المثلثية

$$\sin\theta=\frac{y}{r}=\frac{\text{مقابل}}{\text{وتر}}=\frac{1}{\csc(\theta)}$$

$$\cos\theta=\frac{x}{r}=\frac{\text{مجاور}}{\text{وتر}}=\frac{1}{\sec(\theta)}$$

$$\tan\theta=\frac{y}{x}=\frac{\text{مقابل}}{\text{مجاور}}=\frac{1}{\cot(\theta)}$$

$$1/\sin(\theta)\rightarrow D_{\sin(\theta)}=(-\infty,\infty)=R$$

$$R_{\sin(\theta)}\rightarrow[-1,1]$$

وهي دالة فردية دورية ذات دورة مقدارها

$$2\pi=(2*180)=360$$

$$2/\cos(\theta)\rightarrow D_{\cos(\theta)}=(-\infty,\infty)=R$$

$$R_{\cos(\theta)}\rightarrow[-1,1]$$

وهي دالة زوجية ودورية ذات دورة مقدارها

$$2\pi=(2*180)=360$$

$$3/\tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)}$$

$$D_{\tan(\theta)}=\left[x:,x\in R,x\neq\frac{n\pi}{2},n=\pm1,\pm2,\pm3,......\right]$$

$$R_{\tan(\theta)}\rightarrow(-\infty,\infty)$$

وهي دالة دورية ذات دورة مقدارها(Π)

$$4/\cot(\theta)=\frac{\cos(\theta)}{\sin(\theta)}$$
$$D_{\cot(\theta)}=[x:,x\in R,x\neq n\pi,n=0,\pm1,\pm2,\pm3,......]$$
$$R_{\cot(\theta)}\rightarrow(-\infty,\infty)$$

وهي دالة دورية ذات دورة مقدارها(Π)

$$5/\sec(\theta)=\frac{1}{\cos(\theta)},\cos(\theta)\neq 0$$
$$D_{\sec(\theta)}=\left[x:,x\in R,x\neq\frac{n\pi}{2},n=\pm1,\pm2,\pm3,......\right]$$
$$R_{\sec(\theta)}=R-[-1,1]$$

وهي دالة دورية ذات دورة مقدارها(2Π)

$$6/\csc(\theta)=\frac{1}{\sin(\theta)},\sin(\theta)\neq 0$$
$$D_{\csc(\theta)}=[x:,x\in R,x\neq n\pi,n=0,\pm1,\pm2,\pm3,......]$$
$$R_{\csc(\theta)}=R-[-1,1]$$

وهي دالة دورية

ملاحظة:- بعض القوانين التي يمكن الاستفادة منها لتبسيط الحل:-

$$1/r^2=x^2+y^2$$
$$2/\sin^2(\theta)+\cos^2(\theta)=1$$
$$\sin^2(\theta)=1-\cos^2(\theta)$$
$$\cos^2(\theta)=1-\sin^2(\theta)$$

$$3/\sin(x+2\pi)=\sin(x)$$
$$\cos(x+2\pi)=\cos(x)$$
$$4/\sin(A+B)=\sin A\cos B+\cos A\sin B$$
$$\cos(A+B)=\cos A\cos B-\sin A\sin A$$
$$5/\tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)},\cot(\theta)=\frac{\cos(\theta)}{\sin(\theta)}$$
$$6/\sec(\theta)=\frac{1}{\cos(\theta)},\csc(\theta)=\frac{1}{\sin(\theta)}$$

قوانين اشتقاق الدوال المثلثية الاعتيادية

$$1/\frac{d\sin(u)}{dx}=\cos(u).\frac{du}{dx}$$

$$2/\frac{d\cos(u)}{dx}=-\sin(u).\frac{du}{dx}$$

$$3/\frac{d\tan(u)}{dx}=\sec^2(u).\frac{du}{dx}$$

$$4/\frac{d\cot(u)}{dx}=-\csc^2(u).\frac{du}{dx}$$

$$5/\frac{d\sec(u)}{dx}=\sec(u).\tan(u).\frac{du}{dx}$$

$$6/\frac{d\csc(u)}{dx}=-\csc(u).\cot(u).\frac{du}{dx}$$

قوانين تكامل الدوال المثلثية الاعتيادية

$$1/\int\sin(u)du=-\cos(u)+c$$

$$2/\int\cos(u)du=\sin(u)+c$$

$$3/\int\tan(u)du=\int\frac{\sin(u)}{\cos(u)}=-\ln|\cos(u)|+c$$

$$4/\int\cot(u)du=\int\frac{\cos(u)}{\sin(u)}=\ln|\sin(u)|+c$$

$$5/\int\sec^2(u)du=\tan(u)+c$$

$$6/\int\csc^2(u)du=-\cot(u)+c$$

$$7/\int\sec(u)\tan(u)du=\sec(u)+c$$

$$8/\int\csc(u)\cot(u)du=-\csc(u)+c$$

أمثلة عن اشتقاق الدوال المثلثية

$EX1: y=\sin^2(x)\rightarrow find\,(y')$

$sol:-$

$y=(\sin x\ \ )^2$

$y'=2\sin(x).\cos(x).1=2\sin(x)\cos(x)$

$EX2: y=\tan(x^2)\rightarrow find\,(y')$

$sol:-$

$1/\ y'=\sec^2(x^2)2x=2x\sec^2(x^2)$

$$2/\ y=\tan(x^2)=\frac{\sin(x^2)}{\cos(x^2)}$$

$$y'=\frac{(\cos(x^2))(\cos(x^2))(2x)-(\sin(x^2))(-\sin(x^2))(2x)}{(\cos(x^2))}$$

$$y'=\frac{2x\cos x^2\cos x^2+2x\sin x^2\sin x^2}{(\csc x^2)^2}$$

$$y'=\frac{2x\left[\cos^2 x^2+\sin^2 x^2\right]}{\cos^2(x^2)}=\frac{2x(1)}{\cos^2 x^2}=2x\sec^2 x^2$$

$$EX\,3: y^2 = \cos(x+y) \to find\,(y')$$
$sol:-$
$$2y\frac{dy}{dx} = -\sin(x+y)(1+\frac{dy}{dx})$$
$$2y.\frac{dy}{dx} = -\sin(x+y) - \sin(x+y).\frac{dy}{dx}$$
$$2y.\frac{dy}{dx} + \sin(x+y)\frac{dy}{dx} = -\sin(x+y)$$
$$[2y+\sin(x+y)].\frac{dy}{dx} = -\sin(x+y)$$
$$\frac{dy}{dx} = \frac{-\sin(x+y)}{2y+\sin(x+y)}$$

$$EX\,4: y = \sqrt{\cot(x)+1} \to find\,(y')$$
$sol:-$
$$y = (\cot(x)+1)^{\frac{1}{2}}$$
$$y' = \frac{1}{2}(\cot(x)+1)^{\frac{-1}{2}}(-\csc^2(x)) = \frac{-\csc^2(x)}{2\sqrt{\cot(x)+1}}$$
$$EX\,5: y = \ln(x)\sin(x) \to find\,(y')$$
$sol:-$
$$y' = \ln(x)\cos(x).1 + \sin(x).\frac{1}{x}.1$$
$$y' = \ln(x)\cos(x) + \frac{\sin(x)}{x}$$

$$EX\,6: y = 10^{e^x}.\cos x^2 \to find\,(y')$$
$sol:-$
$$y' = (10^{e^x}).(-\sin(x^2).2x) + (\cos(x^2))(10^{e^x}.\ln(10).e^x.1$$
$$y' = -2x10^{e^x}\sin(x^2) + 10^{e^x}.\ln 10.e^x\cos(x^2)$$

أمثلة عن تكامل الدوال المثلثية

$$EX\,1:\ find\ : y = \int \frac{\sec(x)\tan(x)}{\sec(x)}dx$$
$sol:-$
$$y = \ln|\sec(x)| + c$$

$$EX\,2:\ find\ : y = \int e^x\sec(e^x)\tan(e^x)dx$$
$sol:-$
$$y = \sec(e^x) + c$$

$$EX\,3:\ find\ :y=\int \sec^2(x)\sqrt{\tan(x+1)}\,dx$$

$sol:-$

$$y=\frac{(\tan(x+1))^{\frac{3}{2}}}{\frac{3}{2}}+c$$

$$EX\,4:\ find\ :y=\int 3x^2\cos(x^3)\,dx$$

$sol:-$

$$y=\sin(x^3)+c$$

$$EX\,5:\ find\ :y=\int \frac{\tan(x)}{\cos(x)}\,dx$$

$sol:-$

$$y=\int \tan(x).\frac{1}{\cos(x)}\,dx$$

$$=\int \tan(x)\sec(x)\,dx=\sec(x)+c$$

EX6:- prove that if f(x) = cot(x) then f^{-}(x)=-csc^2(x)

$$f(x)=\cot(x)=\frac{\cos(x)}{\sin(x)}$$

$$f'(x)=\frac{\sin(x)(-\sin(x))-\cos(x).\cos(x)}{(\sin(x))^2}$$

$$f'(x)=\frac{-\sin^2(x)-\cos^2(x)}{\sin^2(x)}=\frac{-(\sin^2(x)+\cos^2(x))}{\sin^2(x)}=\frac{-1}{\sin^2(x)}=-\csc^2(x)$$

EX7:- prove that if f(x) = tan(x) then f^{-}(x)=sec^2 (x)

$$f(x)=\int \cos(2x)\,dx=\frac{1}{2}\int \cos(2x)\,dx$$

$$f'(x)=\frac{1}{2}\sin(2x)+c=\frac{\sin 2x}{2}+c$$

Inverse Trigonometric Functions الدوال المثلثية العكسية(6-5-)

الدوال المثلثية العكسية :-هي الدوال التي يمكن منها استخراج الزاوية من قيمة الدالة المثلثية

$$1/\ y=\sin^{-1}(x)=Arc(\sin(x))$$

$$2/\ y=\cos^{-1}(x)=Arc(\cos(x))$$

$$3/\ y=\tan^{-1}(x)=Arc(\tan(x))$$

$$4/\ y=\sec^{-1}(x)=Arc(\sec(x))$$

$$5/\ y=\csc^{-1}(x)=Arc(\csc(x))$$

$$6/\ y=\cot^{-1}(x)=Arc(\cot(x))$$

$$7/\ y=\sin^{-1}(x)\neq\frac{1}{\sin(x)}$$

قوانين اشتقاق الدوال المثلثية العكسية

$$1/\ y'=\sin^{-1}(x)=Arc(\sin(x))=\frac{1}{\sqrt{1-x^2}}.\frac{dy}{dx}$$

$$2/\ y'=\cos^{-1}(x)=Arc(\cos(x))=\frac{-1}{\sqrt{1-x^2}}.\frac{dy}{dx}$$

$$3/\ y'=\tan^{-1}(x)=Arc(\tan(x))=\frac{1}{1+x^2}.\frac{dy}{dx}$$

$$4/\ y'=\sec^{-1}(x)=Arc(\sec(x))=\frac{1}{|x|\sqrt{x^2-1}}.\frac{dy}{dx}$$

$$5/\ y'=\csc^{-1}(x)=Arc(\csc(x))=\frac{-1}{|x|\sqrt{x^2-1}}.\frac{dy}{dx}$$

$$6/\ y'=\cot^{-1}(x)=Arc(\cot(x))=\frac{-1}{1-x^2}.\frac{dy}{dx}$$

قوانين تكامل الدوال المثلثية العكسية

$$1/\ y=\int\frac{dx}{\sqrt{1-x^2}}=\sin^{-1}(x)+c$$

$$2/\ y=\int\frac{-dx}{\sqrt{1-x^2}}=\cos^{-1}(x)+c$$

$$3/\ y=\int\frac{dx}{1+x^2}=\tan^{-1}(x)+c$$

$$4/\ y=\int\frac{-dx}{1+x^2}=\cot^{-1}(x)+c$$

$$5/\ y=\int\frac{dx}{|x|\sqrt{x^2-1}}=\sec^{-1}(x)+c$$

$$6/\ y=\int\frac{-dx}{|x|\sqrt{x^2-1}}=\csc^{-1}(x)+c$$

أمثلة عن اشتقاق الدوال المثلثية العكسية

$EX1: y=\sin^{-1}(x^2) \rightarrow find\,(y')$

$sol:-$

$$y'=\frac{1}{\sqrt{1-x^2}}.\frac{dy}{dx}=\frac{1}{\sqrt{1-(x^2)^2}}.2x$$

$EX2: y=x^2 con^{-1}(x^2) \rightarrow find\,(y')$

$sol:-$

$$y'=x^2.\frac{-1}{\sqrt{1-(x^2)^2}}.2x+\cos^{-1}(x^2).2$$

$$=\frac{-2x^3}{\sqrt{1-x^4}}+2x\cos^{-1}(x^2)$$

$EX3: y=\cot^{-1}\left(\frac{1}{x}\right) \rightarrow find\,(y')$

$sol:-$

$$y'=\frac{-1}{1+(\frac{1}{x})^2}.\frac{-1}{x^2}=\frac{1}{x^2(1+(\frac{1}{x})^2)}=\frac{1}{x^2(1+\frac{1}{x^2})}$$

$$=\frac{1}{x^2+\frac{x^2}{x^2}}=\frac{1}{x^2+1}$$

$EX4: y=sce^{-1}(e^x) \rightarrow find\,(y')$

$sol:-$

$$y'=\frac{1}{\sqrt{|e^x|\sqrt{e^{2x}-1}}}.e^x=\frac{1}{\sqrt{e^{2x}-1}}$$

$EX5: y=\tan(\sin^{-1}(2x)) \rightarrow find\,(y')$

$sol:-$

$$y'=sce^2(\sin^{-1}(2x)).\frac{1}{\sqrt{1-(2x)^2}}.2$$

أمثلة عن تكامل الدوال المثلثية العكسية

$EX1:\ find:y=\int\frac{1}{x\sqrt{x^2-1}}dx$

$sol:-$

$$y'=\sec^{-1}(x)+c$$

$EX\,2:\ find\ :y=\int\frac{x^2}{\sqrt{1-x^6}}dx$

$sol:-$

$y=\frac{1}{3}\int\frac{3x^2}{\sqrt{1-(x^3)^2}}.dx=\frac{1}{3}\int\frac{dx}{\sqrt{1-x^2}}=\frac{1}{3}.\sin^{-1}(x^3)+c$

ملاحظة:- في حالة الدوال المثلثية العكسية إذا كانت (a) الموجودة بالقانون مربعة فإنها تحمل حليين وكالاتي:-

$1/\int\frac{du}{\sqrt{a^2-u^2}}=\int\frac{dx}{\sqrt{9-x^2}}$

$a^2=9\to a=3$

$u^2=x^2\to u=x$

$\sin^{-1}\frac{u}{a}+c\to\sin^{-1}\frac{x}{3}+c$

$\cos^{-1}\frac{u}{a}+c\to\cos^{-1}\frac{x}{3}+c$

$2/\int\frac{du}{a^2+u^2}=\int\frac{dx}{9+x^2}$

$a^2=25\to a=5$

$u^2=x^2\to u=x$

$\frac{1}{a}\tan^{-1}\frac{u}{a}+c\to\frac{1}{5}\tan^{-1}\frac{x}{5}+c$

$\frac{1}{a}\cot^{-1}\frac{u}{a}+c\to\frac{1}{5}\cot^{-1}\frac{x}{5}+c$

أمثلة على القاعدة أعلاه

$EX1:\ find\ :y=\int\frac{1}{x(3+\ln^2x)}dx$

$sol:-$

$a^2=3\to a=\sqrt{3}$

$u^2=\ln^2(x)\to u=\ln(x)$

$\frac{1}{\sqrt{3}}\tan^{-1}(\frac{\ln(x)}{\sqrt{3}})+c$

$\frac{1}{\sqrt{3}}\cot^{-1}(\frac{\ln(x)}{\sqrt{3}})+c$

$EX\,2:\ find\ :y=\int\frac{1}{\sqrt{4-x^2}}dx$

$sol:-$

$a^2=4\to a=2$

$u^2=x^2\to u=x$

$$= \sin^{-1}\frac{x}{2} + c$$

$$= \cos^{-1}\frac{x}{2} + c$$

$$EX\,3:\ find\ : y = \int \frac{2\cos(x)}{1+\sin^2(x)}dx$$

$sol:-$

$a^2 = 1 \rightarrow a = 1$

$u^2 = \sin^2 x \rightarrow u = \sin x \rightarrow du = \cos x dx$

$$\int \frac{2du}{1+u^2} = 2\int \frac{du}{1+u^2} = 2\tan^{-1}(x) + c$$

(6-6)الدوال المثلثية الزائدية

HyperbolicTrigonometric Functions

يمكن التعبير عن الدوال الزائدية باستخدام الدالة الآسية وسميت بالدوال الزائدية لان دالتين الجيب والجيب تمام تمثل معادلة قطع زائدي وتكتب بالشكل الأتي:-

$$1/\ y = \sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$D_{\sinh x} = (-\infty, \infty), R_{\sinh x} = (-\infty, \infty)$$

$$2/\ y = \cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$D_{\cosh x} = (-\infty, \infty), R_{\cosh x} = [1, \infty)$$

$$3/\ y = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$D_{\tanh x} = (-\infty, \infty), R_{\tanh x} = \left[-1,1\right]$$

$$4/\ y = \sec h(x) = \frac{1}{\cos(x)} = \frac{2}{e^x + e^{-x}}$$

$$D_{\sec hx} = (-\infty, \infty), R_{\sec hx} = (0,1]$$

$$5/\ y = \csc h(x) = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

$$D_{\csc hx} = (-\infty,0) and (0, \infty), R_{\csc hx} = (-\infty,0) and (0, \infty)$$

$$6/\ y = \coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$D_{\coth x} = (-\infty,0) and (0, \infty)$$

$$R_{\coth x} = (-\infty,-1) and (1, \infty)$$

قوانين يمكن الاستفادة منها بتبسيط الحل

$$1/\cosh^2 x - \sin^2 hx = 1$$

$$2/\tanh^2 x + \sec h^2 x = 1$$

$$3/\coth^2 x - \csc h^2 x = 1$$

$$4/\ coh^2 x = \frac{1+\cosh 2x}{2}$$

$$5/\sinh^2 x = \frac{\cosh 2x - 1}{2}$$
$$6/\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$
$$7/\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

قوانين اشتقاق الدوال المثلثية الزائدية

$$1/\frac{d \sinh u}{dx} = \cosh u.\frac{du}{dx}$$
$$2/\frac{d \cosh u}{dx} = \sinh u.\frac{du}{dx}$$
$$3/\frac{d \tanh u}{dx} = \sec h^2 u.\frac{du}{dx}$$
$$4/\frac{d \sec hu}{dx} = -\sec hu.\tanh u.\frac{du}{dx}$$
$$5/\frac{d \csc hu}{dx} = -\csc hxu.\coth u.\frac{du}{dx}$$
$$6/\frac{d \coth u}{dx} = -\csc h^2 u.\frac{du}{dx}$$

قوانين تكامل الدوال المثلثية الزائدية

$$1/\int \cosh u du = \sinh u + c$$
$$2/\int \sinh u du = \cosh u + c$$
$$3/\int \sec h^2 u du = \tanh u + c$$
$$4/\int \csc h^2 u du = -\coth u + c$$
$$5/\int \sec hu.\tanh u.du = -\sec hu + c$$
$$6/\int \csc hu \coth u.du = -\csc hu + c$$

أمثلة عن اشتقاق الدوال المثلثية الزائدية

$EX1: y = \sinh x^2 \rightarrow find\,(y')$

$sol:-$

$y' = \cosh x^2.2x = 2x \cosh x^2$

$EX2: y = \tanh(\ln x^3) \rightarrow find\,(y')$

$sol:-$

$$y' = \sec^2 h(\ln(x^3).\frac{1}{x^3}.3x^2 = \frac{3\sec^2 h(\ln x^3)}{x}$$

$EX\,3: y=\sec h(e^{x^2}) \rightarrow find\,(y')$

$sol:-$

$y'=-sch(e^{x^2}).\tanh(e^{x^2}).e^{x^2}.2x$

$=-2xe^{x^2}\sec h(e^{x^2})\tanh(e^{x^2})$

$EX\,4: y=\coth(\sec^{-1}x^2) \rightarrow find\,(y')$

$sol:-$

$$y'=-\csc^2 h(\sec^{-1}x^2).\frac{1}{x^2\sqrt{x^4-1}}.2x=\frac{-2\csc^2 h(\sec^{-1}x^2)}{x\sqrt{x^4-1}}$$

$EX\,5: y=\sin x\tanh(x^3) \rightarrow find\,(y')$

$sol:-$

$y'=\sin(x).\sec^2 h(x^3).3x^2+\tanh h(x^3).\cos x$

$=3x^2\sin x\sec^2 h(x^3)+\cos x\tanh(x^3)$

$$EX\,6: y=\frac{\ln(x)}{\tanh(x^2)} \rightarrow find\,(y')$$

$sol:-$

$$y'=\frac{(\tanh(x^2).\frac{1}{x}.1)-(\ln x.\sec^2 h(x^2)2x}{(\tanh(x^2))^2}$$

$$=\frac{\frac{\tanh(x^2)}{x}-2x\ln(x)\sec^2 h(x^2)}{(\tanh(x^2))^2}$$

أمثلة عن التكامل الدوال المثلثية الزائدية

$EX\,1:\ find\ : y=\int\tanh^2(x)dx$

$sol:-$

$\tanh^2(x)+\sec h^2(x)=1$

$\tanh^2(x)=1-\sec h^2(x)$

$\int(1-\sec h^2(x))dx \rightarrow \int 1dx-\int\sec^2 hdx=x-\tanh(x)+c$

$$EX\,2:\ find\ : y=\int\sqrt{\frac{\cosh(x)-1}{2}}dx$$

$sol:-$

$$y=\int\sqrt{\sinh^2\frac{x}{2}}dx=\int\sinh\frac{x}{2}dx=2\cosh\frac{x}{2}$$

$EX\,3:\ find\ :y=\int \coth^2 dx$

$sol:-$

$y=\coth^2 x-\csc h^2 x=1$

$\coth^2 x=1+\csc h^2 x$

$\int(1+\csc h^2 x)dx=\int 1dx+\int \csc h^2 xdx$

$=x+(-\coth x)+c=x-\coth x+c$

$EX\,4:\ find\ :y=\int \sec hx.\tanh x.e^{\sec hx}dx$

$sol:-$

$y=-e^{\sec hx}+c$

(7-6) الدوال المثلثية العكسية الزائدية

Hyperbolic Inverse Trigonometric Functions

$1/\ y=\sinh^{-1}(x)=\ln(x+\sqrt{x^2+1})$

$D_{\sinh^{-1}x}=(-\infty,\infty), R_{\sinh^{-1}x}=(-\infty,\infty)$

$2/\ y=\cosh^{-1}(x)=\ln(x+\sqrt{x^2-1})$

$D_{\cosh^{-1}x}=[1,\infty), R_{\cosh^{-1}x}=[0,\infty)$

$3/\ y=\tanh^{-1}(x)=\frac{1}{2}\ln\frac{1+x}{1-x}$

$D_{\tanh^{-1}x}=(-1,1), R_{\tanh^{-1}x}=(-\infty,\infty)$

$4/\ y=\sec h^{-1}(x)=\frac{\ln(1+\sqrt{1-x^2}}{x}$

$D_{\sec h^{-1}x}=(0,1], R_{\sec h^{-1}x}=[0,\infty)$

$5/\ y=\csc h^{-1}(x)=\frac{\ln(1+\sqrt{1+x^2}}{|x|}$

$D_{\csc h^{-1}x}=(-\infty,0)and(0,\infty), R_{\csc hx}=(-\infty,0)and(0,\infty)$

$6/\ y=\coth^{-1}(x)=\frac{1}{2}\ln\frac{x+1}{x-1}$

$D_{\coth^{-1}x}=(-\infty,-1)and(1,\infty)$

$R_{\coth^{-1}x}=(-\infty,0)and(0,\infty)$

قوانين عن اشتقاق الدوال المثلثية الزائدية العكسية

$1/\ y'=\sin^{-1}hu=\frac{1}{\sqrt{u^2+1}}.\frac{du}{dx}$

$2/\ y'=\cos^{-1}hu=\frac{1}{\sqrt{u^2-1}}.\frac{du}{dx}$

$3/\ y'=\tan^{-1}hu=\frac{1}{1-u^2}.\frac{du}{dx}$

$$4/\ y' = \sec^{-1} hu = \frac{-1}{u\sqrt{1-u^2}}.\frac{du}{dx}$$

$$5/\ y' = \csc^{-1} hu = \frac{-1}{|u|\sqrt{1+u^2}}.\frac{du}{dx}$$

$$6/\ y' = \cot^{-1} hu = \frac{1}{1-u^2}.\frac{du}{dx}$$

ملاحظة:-

$$1/\ \tanh^{-1} u = \cot^{-1} h\frac{1}{u}$$

$$2/\ \sec h^{-1} u = \cos^{-1} h\frac{1}{u}$$

$$3/\ \csc h^{-1} u = \sinh^{-1}\frac{1}{u}$$

أمثلة عن اشتقاق الدوال المثلثية الزائدية العكسية

$EX1: y = \cosh^{-1}(\sec e^{3x}) \rightarrow find\,(y')$

$sol:-$

$$y' = \frac{1}{\sqrt{((\sec(e^{3x}))^2 - 1}}.\sec(e^{3x})\tan(e^{3x}).3e^{3x}$$

$$= \frac{3e^{3x}}{\sqrt{\tan^2(e^{3x})}}.\sec(e^{3x})\tan(e^{3x}) = \frac{3e^{3x}}{\tan(e^{3x})}.\sec(e^{3x}).\tan(e^{3x})$$

$$= 3e^{3x}.\sec(e^{3x})$$

$EX2: y = \sec h^{-1}(e^{-x}) \rightarrow find\,(y')$

$sol:-$

$$y' = \frac{-1}{e^{-x}\sqrt{1-(e^{-x})^2}}.-e^{-x} = \frac{-e^{-x}}{e^{-x}\sqrt{1-(e^{-x})^2}} = \frac{1}{\sqrt{1-e^{-2x}}}$$

$EX3: y = \sin^{-1} x^2.\tan^{-1} x^2 \rightarrow find\,(y')$

$sol:-$

$$y' = \sin^{-1} x^2.\frac{1}{1+x^2} + \tanh^{-1} x.\frac{1}{\sqrt{1-(x^2)^2}}.2x$$

$$y' = \frac{\sin^{-1} x^2}{1+x^2} + \frac{2x\tanh^{-1} x}{\sqrt{1-x^4}}$$

$EX4: y^2 = \coth^{-1}(xy) \rightarrow find\,(y')$

$sol:-$

$$2y.\frac{dy}{dx} = \frac{1}{1-(xy)^2}.(x.\frac{dy}{dx} + y\frac{dx}{xd})$$

$$2y.\frac{dy}{dx} = \frac{1}{1-(xy)^2}(x.\frac{dy}{dx} + y)$$

$$2y.\frac{dy}{dx}=\frac{x}{1-(xy)^2}.\frac{dy}{dx}+\frac{y}{(1-(xy)^2)}$$

$$(2y-\frac{x}{1-(xy)^2})\frac{dy}{dx}=\frac{y}{1-(xy)^2}$$

$$\frac{dy}{dx}=\frac{1-(xy)^2}{2y-\frac{x}{1-(xy)^2}}$$

قوانين عن تكامل الدوال المثلثية الزائدية العكسية

$$1/\int\frac{du}{\sqrt{u^2+1}}=\sinh^{-1}u+c$$

$$2/\int\frac{du}{\sqrt{u^2-1}}=\cosh^{-1}u+c$$

$$3/\int\frac{du}{1-u^2}=\begin{Bmatrix}\tanh^{-1}u+c,|u|\prec 1\\ \coth^{-1}u+c,|u\succ 1|\end{Bmatrix}$$

$$4/\int\frac{du}{u\sqrt{1-u^2}}=\sec h^{-1}u+c=\cos^{-1}h\frac{1}{u}+c$$

$$5/\int\frac{du}{u\sqrt{1+u^2}}=\csc h^{-1}u+c=\sinh^{-1}\frac{1}{u}+c$$

أمثلة عن تكامل الدوال المثلثية الزائدية العكسية

$$EX1:\ find\ :y=\int\frac{dx}{\sqrt{1+4x^2}}$$

$sol:-$

$$y=\frac{1}{2}\int\frac{2dx}{\sqrt{1+(2x)^2}}=\frac{1}{2}\sinh^{-1}2x+$$

$$EX2:\ find\ :y=\int\frac{3x^2e^{x^3}dx}{1-e^{2x^3}}$$

$sol:-$

$$y=\int\frac{3x^2e^{x^3}dx}{1-(e^{x^3})^2}=\tanh^{-1}(e^{x^3})+c$$

ملاحظة :- في حالة كون الثابت رقم (1) الموجود بالقوانين التابع للتكاملات الدوال المثلثية الزائديه العكسية لو استبدل بأي ثابت أخر سوف تكون القوانين كالأتي:-

$$1/\int\frac{du}{u^2+a^2}=simh^{-1}\frac{u}{a}+c$$

$$2/\int\frac{du}{u^2-a^2}=\cosh^{-1}\frac{u}{a}+c$$

$$3/\int\frac{du}{u\sqrt{a^2+u^2}}=\frac{-1}{a}\csc h^{-1}\frac{|u|}{a}+c$$

$$4/\int \frac{du}{u\sqrt{a^2-u^2}} = \frac{1}{a}\operatorname{sec}h^{-1}\frac{|u|}{a}+c$$

$$5/\int \frac{du}{a^2-u^2} = \frac{1}{a}\tanh^{-1}\frac{u}{a}+c \rightarrow |u| \prec a$$

$$6/\int \frac{du}{a^2-u^2} = \frac{1}{a}\coth^{-1}\frac{u}{a}+c \rightarrow |u| \succ a$$

أمثلة

$$1/\ y=\int \frac{dx}{x\sqrt{x^2+4}}$$

$sol:-$

$$y=\int \frac{dx}{x\sqrt{x^2+(2)^2}} = -\frac{1}{a}\operatorname{csc}h^{-1}\frac{|x|}{a}+c$$

$$y=\frac{-1}{2}\operatorname{csc}h^{-1}\frac{|x|}{2}+c$$

$$EX\ 2/\ y=\int \frac{dx}{\sqrt{4+x^2}}$$

$sol:-$

$$y=\int \frac{dx}{\sqrt{(2)^2+x^2}} = \sinh^{-1}\frac{x}{a}+c$$

$$y=\sinh^{-1}\frac{x}{2}+c$$

الزاوية	x=0	x=90	x=180	x=270	x=360	x=30	x=60	x=45
sin(x)	0	1	0	-1	0	1/2	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
cos(x)	1	0	-1	0	1	$\frac{\sqrt{3}}{2}$	1/2	$\frac{1}{\sqrt{2}}$
tan(x)	0	$\pm\infty$	0	$\pm\infty$	0	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1
cot(x)	$\pm\infty$	0	$\pm\infty$	0	$\pm\infty$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	1
sec(x)	1	$\pm\infty$	-1	$\pm\infty$	1	$\frac{2}{\sqrt{3}}$	2	$\sqrt{2}$
csc(x)	$\pm\infty$	1	$\pm\infty$	-1	$\pm\infty$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$

** الواجب**

السؤال الأول:- جد المشتقة للدوال الآتية:-

$1/\ y = e^{x\ln(x)}$

$2/\ y = \log_7 \sin(x)$

$3/\ y = \log_{16}(\log_3(x))$

$4/\ y = \log_5(\sqrt{x^2 - 1})$

$5/\ y = e^{x^2}.\ln(\sin(x))$

$6/\ y = x^{\sin x}$

$7/\ y = (x^2 + 1)^x$

$8/\ y = \dfrac{x}{\csc^{-1}(x^2)}$

$9/\ y = \sec^{-1}(5^{x^2})$

$10/\ y = \sqrt{\csc h(e^x)}$

$11/\ y = 15^{\sec h(\ln x)}$

$12/\ y = \ln(x^3 \tanh x)$

$13/\ y = \sin(x + \cosh x)$

$14/\ y = \dfrac{e^x}{\sec h^{-1}(e^x)}$

$15/\ y = Arc\csc h(\sinh^{-1} x)$

السؤال الثاني:- جد التكامل للدوال الآتية:-

$1/\ y = \int \left(\dfrac{e^x}{e^x - 1} - \dfrac{1}{x} \right) dx$

$2/\ y = \int 35^{x^3}.x^2 dx$

$3/\ y = \int \cos(x).10^{\sin(x)} dx$

$4/\ y = \int 8^{(x^2+2x)}.(x+1) dx$

$5/\ y = \int_1^2 2^{2x} dx$

السؤال الثالث:- برهن ما يأتي:-

$1/\ y = \log_e(x) = \ln(x)$

$2/\ a^x = e^{x(\ln a)}$

$3/\ y' = y, y = e^x$

$4/\ \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \dfrac{\cos(x)}{\sin(x^2)} dx = -2$