الفصل الرابع

المتتابعات والمتسلسلات

Sequences and Series

The sequence المتتابعة (1-4)

المتتابعة:- وهي عبارة عن دالة منطلقها جميع الإعداد الطبيعية الموجبة والمدى لها مجموعة الإعداد الحقيقية وتأخذ الصيغة الآتية:-

$a_n = a(n) = <a_n> = \{a_n\} =$

إي تعبير حسابي يحتوي على (n)

Remark:-

1 we of ton call (a_n) the (n) the term of the sequence

2 The sequences divided two:-

a/The sequence is finite.

b/ The sequence is in finite.

$$\{a_n\}_{n=1}^{10} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}\} = finite.sequence$$

$$\{a_n\}_{n=1}^{\infty} = \{a_1, a_2, a_3, ...\} = \inf inite.sequence$$

EX1:- Find the sequence of

$$1 - \{n\}_{n=1}^{\infty}$$

$$2 - \left\{\frac{1}{n}\right\}_{n=1}^{\infty}$$

$$1 - a_n = \{n\}_{n=1}^{\infty}$$

$$a_1 = 1$$

$$a_2 = 2$$

$$a_3 = 3$$

:

:

$$a_n = \prec 1, 2, 3, ... \succ$$

$$2 - a_n = \left\{\frac{1}{n}\right\}_{n=1}^{\infty}$$

$$a_1 = \frac{1}{1}$$

$$a_2 = \frac{1}{2}$$

$$a_3 = \frac{1}{3}$$

$$a_4 = \frac{1}{4}$$

:

:

$$a_n = \prec 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ... \succ$$

EX2:- Find the sequence of

$$\{n^2\}_{n=1}^{5}$$

$sol:-$

$$an=\{n^2\}_{n=1}^{5}$$

$$a_1=(1)^2=1$$

$$a_2=(2)^2=4$$

$$a_3=(3)^2=9$$

$$a_4=(4)^2=16$$

$$a_5=(5)^2=25$$

$$a_n=\prec 1,4,9,16,25\succ$$

EX3:-if the a_n<1,2,3,4,5> defined by function of sequence

$sol:-$

$$an=f(n)=n$$

$$\{a_n\}_{n=1}^{5}=\{n\}_{n=1}^{5}$$

$$a_1=f(1)=1$$

$$a_2=f(2)=2$$

$$a_3=f(3)=3$$

$$a_4=f(4)=4$$

$$a_5=f(5)=5$$

EX4:-if the (a_n) defined by function of sequence

$$a_n=\prec 1,\sqrt{2},\sqrt{3},2,\sqrt{5},\sqrt{6},\sqrt{7},\sqrt{8},3\succ$$

Sol:-

$$a_n=f(n)=\sqrt{n}$$

$$\{a_n\}_{n=1}^{\infty}=\{\sqrt{n}\}_{n=1}^{\infty}$$

ملاحظة:- عند رسم أي متتابعة فان x=n وان $y=a_n$.

Increasing and Decreasing of sequence المتتابعة المتزايدة والمتناقصة (2-4)

1 An Increasing sequence $\{a_n\}$:- is a sequence for which $a_{n+1}\geq a_n$ for all (n) but we say the strictly increasing sequence $\{a_n\}$ is a sequence for which $a_{n+1}>a_n$ for all(n).

2 AD creasing sequence $\{a_n\}$:- is a sequence for which $a_{n+1}\leq a_n$ for all (n) but we say the strictly d creasing sequence $\{a_n\}$ is a sequence for which $a_{n+1}<a_n$ for all(n).

3 A monotonic sequence:- is a sequence which is either in ceasing or decreasing

EX:

$a_n = \{1,1,2,2,4,4,32,32,\ldots\}$

هذه المتتابعة حققت الشرط $a_{n+1} \geq a_n$ إذن increasing sequence

$a_n = \{1, 2,2,3,3,3,4,4,4,4,5,5,5,5,5,\ldots\ \ldots\}$

هذه المتتابعة حققت الشرط $a_{n+1} \geq a_n$ إذن increasing sequence

$a_n = \{2,4,8,16,32,\ldots\}$

هذه المتتابعة حققت الشرط $a_{n+1} > a_n$ إذن strictly increasing sequence

$a_n = \{2,3,5,7,11,\ldots\}$

هذه المتتابعة حققت الشرط $a_{n+1} > a_n$ إذن strictly increasing sequence

$a_n = \{5,5,4,4,3,3,2,2,1,1,0,0,\ldots\}$

هذه المتتابعة حققت الشرط $a_{n+1} \leq a_n$ إذن decreasing sequence

$a_n = \{-2,-4,-6,-8,-10,-12,\ldots\}$

هذه المتتابعة حققت الشرط $a_{n+1} < a_n$ إذن strictly decreasing sequence

$a_n = \{1,-1,1,-1,1,-1,\ldots\}$

a_n= neither increasing and decreasing

(3-4) المتتابعة المقيدة Bounds of sequences

1/ If there exists a constant (h) such that $a_n \leq h$ for all values of then the sequence $\{a_n\}$ is said to be bounded upper.

2/ If there exists a constant (h) such that $a_n \geq h$ for all values of then the sequence $\{a_n\}$ is said to be bounded down.

3/ we say of any sequence is bounded if have upper and down bounded.

ملاحظة1:- إذا كانت المتتابعة مقيدة من الأعلى فقط أو مقيدة من الأسفل فقط بشكل عام لا تعتبر مقيدة .

ملاحظة 2:-

$$1-\lim_{n\to\infty}\left(\frac{1}{n}\right)=0$$

$$2-\lim_{n\to\infty} c = c$$

$$3-\lim_{n\to\infty} x^n = \infty, x \succ 1$$

$$4-\lim_{n\to\infty} x^n = 0, 0 \prec |x| \prec 1$$

(4-4) المتتابعة المتقاربة والمتباعدة Converge and Diverge of sequences

sequences $\{a_n\}$ is said to be convergent if $\{a_n\}$ has limit and

$\lim_{n\to\infty} a_n$ is exist but we say the sequence is diverge when the

$\lim_{n\to\infty} a_n$ dose not exists

EX1:- Is $a_n=n$ converge or diverge

Sol:-

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n = \infty$$

The limit does not exists the (a_n) is diverge

EX2:- Is $a_n=1/n$ converge or diverge

Sol:-

$$a_n = 1/n$$

$$\{a_n\} = \{1, 1/2, 1/3, 1/4, ...\}$$

$$\lim_{n\to\infty}(1/n) = 1/\infty = 0$$

The limit exists the (a_n) is converge

EX3:- Is $a_n=(3+2n/5+n)$ converge or diverge

Sol:-

$$\lim_{n\to\infty} \frac{3+2n}{5+n} = \frac{\frac{3}{n}+\frac{2n}{n}}{\frac{5}{n}+\frac{n}{n}} = \lim_{n\to\infty} \frac{\frac{3}{n}+2}{\frac{5}{n}+1}$$

$$\frac{\frac{3}{\infty}+2}{\frac{5}{\infty}+1} = \frac{0+2}{0+1} = 2$$

The limit exists the (a_n) is converge

مبرهنة (5-4)

If the sequence of real number $\{a_n\}$ is convergent then $\{a_n\}$ bounded

تقارب $\rightarrow$ مقيدة

مقيدة $\xrightarrow{\text{لايودي}}$ تقارب

ولإثبات النظرية أعلاه نأخذ المثال الأتي:-

EX1:- Is $a_n=1/n$ converge or diverge

Sol:-

$$\lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$$

The limit exists , a_n convergent

$$a_n = \left\{\frac{1}{n}\right\}_{n=1}^{\infty}$$

$$a_1 = 1$$

$$a_2 = \frac{1}{2}$$

$$a_3 = \frac{1}{3}$$

$$a_4 = \frac{1}{4}$$

:

:

$$a_\infty = \frac{1}{\infty} = 0$$

The upper bounded =1, the lower bounded =0 the sequences bounded.

ملاحظة:- كل متتابعة من النوع المتذبذب تأخذ قيم موجبة ثم قيم سالبة ليست لها غاية.

EX1:- Find the convergent or divergent , bounded or un bounded , incersing or decersing of the sequence

$$\left\{\frac{n+1}{n^2}\right\}_{n=1}^{\infty}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+1}{n^2} = \lim_{n\to\infty} \frac{\frac{n}{n^2}+\frac{1}{n^2}}{\frac{n^2}{n^2}} = \lim_{n\to\infty} \frac{\frac{1}{n}+\frac{1}{n^2}}{1} =$$

$$= \frac{\frac{1}{\infty}+\frac{1}{(\infty)^2}}{1} = \frac{0+0}{1} = 0$$

The limit exists =0 the sequence is convergent

$$a_n = \left\{\frac{n+1}{n^2}\right\}_{n=1}^{\infty}$$

$$a_1 = \frac{1+1}{1^2} = 2$$

$$a_2 = \frac{2+1}{(2)^2} = 0.75$$

$$a_3 = \frac{3+1}{(3)^2} = 0.44$$

$$a_4 = \frac{4+1}{(4)^2} = 0.31$$

$$a_n = \prec 2, 0.75, 0.44, 0.31, ... \succ$$

The sequence is bounded of upper =2 but does not bounded of lower (down) the sequences does not bounded

$$a_n = \prec 2, 0.75, 0.44, 0.31 \succ$$

$$a_{n+1} \prec a_n$$

A strictly decreasing sequence

EX2:- Find the convergent or divergent , bounded or un bounded , incensing or decreasing of the sequence

$$a_n = \{(-1)^n\}_{n=1}^{\infty}$$

$sol:-$

$$a_n = \{(-1)^n\}_{n=1}^{\infty}$$
$$a_1 = (-1)^1 = -1$$
$$a_2 = (-1)^2 = 1$$
$$a_3 = (-1)^3 = -1$$
$$a_4 = (-1)^4 = 1$$
:
:
$$\prec a_n \succ = \prec -1,1,-1,1,-1,1,...... \succ$$

The (a_n) is either increasing or decreasing the $a_n = (-1)^n$ is monotonic

$$(-1)^n = \begin{cases} +1, n.......even \\ -1, n........odd \end{cases}$$

وحسب النظرية إن كل متتابعة متذبذبة لا تملك غاية

The $a_n=(-1)^n$ is divergent

The $a_n = (-1)^n$ is bounded because has upper bounded = 1, lower bounded =-1

Series متسلسلات (6-4)

المتسلسلات:- وهي عبارة عن مجموع حدود المتتابعة (المتسلسلة) والتي يرمز لها بالرمز

$$S_n = \sum_{n=1}^{\infty} a_n$$
$$s_1 = a_1$$
$$s_2 = a_1 + a_2$$
$$s_3 = a_1 + a_2 + a_3$$
$$s_4 = a_1 + a_2 + a_3 + a_4$$
:
:
$$s_n = a_1 + a_2 + a_3 + ... + a_n$$

حيث إن a_n يدعى بالحد النوني.
S_n يدعى بالمجموع الجزئي.
$\{s_n\}_{n=1}^{\infty}$ يدعى متتابعة المجاميع الجزئية.

تقسم المتسلسلات إلى قسمين:-

$$1/\ finite \rightarrow S_n = \sum_{n=1}^{m} a_n$$

$$2/\inf inite \rightarrow S_n = \sum_{n=1}^{\infty} a_n$$

حيث إن (m) هو أي عدد

$$EX1:-find.the.serie, S_n = \sum_{n=1}^{4} \frac{1}{n}$$

$sol:-$

$$s_1 = \frac{1}{1} = 1$$

$$s_2 = 1 + \frac{1}{2}$$

$$s_3 = 1 + \frac{1}{2} + \frac{1}{3}$$

$$s_4 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$$

$$EX2:-find.the.serie, S_n = \sum_{n=0}^{\infty} n^2 + 1$$

$sol:-$

$$s_0 = (0)^2 + 1 = 1$$

$$s_1 = 1 + ((1)^2 + 1) = 3$$

$$s_2 = 3 + ((2)^2 + 1) = 8$$

$$s_3 = 8 + ((3)^2 + 1) = 18$$

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:

(7-4) تقارب وتباعد المتسلسلات

يتم اختبار تقارب وتباعد المتسلسلات وذلك من خلال اختبار المتتابعة الأصلية للمتسلسلة فإذا متقاربة تعتبر المتسلسلة متقاربة أيضا أما إذا كانت المتتابعة الأصلية للسلسة متباعدة نعتبر المتسلسلة متباعدة أيضا.

EX1:- Find is series convergent or divergent of

$$S_n = \sum_{n=0}^{\infty} n^2$$

Sol:-

$$S_n = \sum_{n=0}^{\infty} n^2 \Rightarrow a_n = n^2$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n^2 = (\infty)^2 = \infty$$

The sequence (a_n) is divergent

The series (S_n) is divergent

EX2:- Find is series convergent or divergent of

$$S_n = \sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$$

$Sol:-$

$$a_n = \frac{\ln(n)}{n^2} = \lim_{n\to\infty} \frac{\ln(n)}{n^2} = \frac{\ln(\infty)}{(\infty)^2} = \frac{\infty}{\infty} \to Lopital.Rule$$

$$\stackrel{*}{=} \lim_{n\to\infty} \frac{\frac{1}{n}}{2n} = \lim_{n\to\infty} \frac{1}{2n^2} = \frac{1}{2(\infty)^2} = \frac{1}{\infty} = 0$$

The sequence (a_n) is convergent
The series (S_n) is convergent

(8-4) أنواع المتسلسلات

1 The Arithmetic progression
المتسلسلة العددية

Def:- Arithmetic progression:- is series whose (n) the term is

a_n = (a +(n-1) d) where (a) is constant (d) is called the common difference of the series the (n) the partial sum (s_n) of arithmetic progression.

S_n= n/2 [2a +(n-1) d]

ملاحظة:- وان المتتالية العددية تنتج دائما من إضافة أو طرح رقم معين على الحد الأول.

Ex1:- Find the (n th) partial sum of the series (5,+2,-1,-4,-7,-10,-13,…..)
Sol:-
The series is arithmetical progression a=5, d=-3
S_n= n/2 [2a +(n-1) d]
S_n = n/2 [2(5)+ (n-1) (-3)]
S_n = n/2[10-3n+3]
S_n = n/2 [13-3n]
a_n = [a+ (n-1) d] = [5+(n-1) (-3)]

2 The Geometric progression
المتسلسلة الهندسية

Geometric progression:- is series whose (n th) term a_n =ar^{n-1} where a , and r is constant to the (n th) partial sum (S_n) of a geometric progression

$1-(r) \neq 1, |r| \succ 1$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$2-|r| = 1$

$S_n = na$

$3-|r| \prec 1$

$$S_n = \frac{a}{1-r}$$

ملاحظة:- دائما المتسلسلة الهندسية تنتج من حاصل ضربها أو قسمتها على الثابت.

EX1:-Find the (n th) partial sum of the series (1-2+4-8+16-32)

الحل:- من ملاحظة المتسلسلة أعلاه نجد أنها تم الحصول عليها من ضربها في الرقم (2-) فنستنتج إن المتسلسلة من النوع الهندسية. حيث إن a = 1 تمثل الحد الأول أما القيمة التي ضربت بها r = -2

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{1(1-(-2)^n)}{1-(-2)} = \frac{1-(-2)^n}{1+2} = \frac{1-(-2)^n}{3}$$

$$a_n = ar^{n-1} = (1)(-2)^{n-1}$$

EX2:- If the series convergent or divergent

$$S_n = \sum_{n=1}^{\infty} \frac{3}{10^n}$$

Sol:-

$$S_n = \sum_{n=1}^{\infty} \frac{3}{10^n} = \frac{3}{10^1} + \frac{3}{10^2} + \frac{3}{10^3} + \dots\dots\dots\dots + \frac{3}{10^n}$$

$$= \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \dots\dots\dots + \frac{3}{10^n}$$

$$a = \frac{3}{10}, r = \frac{a_2}{a_1} = \frac{\frac{3}{100}}{\frac{3}{10}} = \frac{3}{100} \times \frac{10}{3} = \frac{10}{100} = \frac{1}{10}$$

$$r = \left|\frac{1}{10}\right| \prec 1$$

$$\therefore S_n convergent$$

**** اشتقاق الصيغة العامة للمتسلسلة الهندسية****

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \dots\dots\dots + ar^{n-1}$$

$$S_n = a + ar + ar^2 + ar^3 + \dots\dots\dots + ar^{n-1}$$

$$rS_n = ar + ar^2 + ar^3 + ar^4 + \dots\dots\dots + ar^{n-1} + ar^{n-1+1}$$

$$S_n - rS_n = a - ar^n$$

$$S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

ملاحظة:-

$$|r| = 1 \Rightarrow S_n \to divergent$$

$$|r| \succ 1 \Rightarrow S_n \to divergent$$

$$|r| \prec 1 \Rightarrow S_n \to convergent$$

3 The Harmonic series

المتسلسلة التوافقية

Whose (n th) term is (1/n) is called the harmonic series:-

$$S_n = \sum_{n=1}^{\infty} \frac{1}{n}$$

4 The AL-ternating series

المتسلسلات المتناوبة

If $a_n > 0$ for each (n) the series is called an AL-ternating series

$$S_n = \sum_{n=1}^{\infty} (-1)^{n+1}$$

5 The P- series

P متسلسلة

Given any number (p) the p- series is the series:-

$$S_n = \sum_{n=1}^{\infty} \frac{1}{n^p}$$

6 The General series

المتسلسلة العامة

$$S_n = \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

All the (3-6) are not have (n th) partial sum of series

Theorem

إذا كانت لدينا متسلسلتان فانه:-

$$\sum_{n=1}^{\infty} a_n , \sum_{n=1}^{\infty} b_n$$

$$if . s_n = \sum_{n=1}^{\infty} (a_n \pm b_n) = \sum_{n=1}^{\infty} a_n \pm \sum_{n=1}^{\infty} b_n$$

$$1 \sum_{n=1}^{\infty} a_n convergent, \sum_{n=1}^{\infty} b_n convergent$$

$$\therefore S_n = convergent$$

$$2 \sum_{n=1}^{\infty} a_n divergent, \sum_{n=1}^{\infty} b_n , convergent$$

$$\therefore S_n = divergent$$

$$3 \sum_{n=1}^{\infty} a_n convergent, \sum_{n=1}^{\infty} b_n divergent$$

$$\therefore S_n = divergent$$

7 Power series

سلاسل القوى

وهي عبارة عن متسلسلة الأساس لها (x) والأس لها (n) وتأخذ الشكل الأتي:-

$$S_n = \sum_{n=0}^{m-1} a_n x^n$$

$$S_n = a_0 x^0 + a_1 x^1 + a_2 x^2 + ... + a_n x^n$$

$$S_n = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$$

هنالك نوعيين من متسلسلة القوى

**Tylor 's – series

متسلسلة تايلر

$$f(x) = S_n = f(a) + \frac{f'(a)(x-a)^1}{1!} + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + ... + \frac{f^n(a)(x-a)^n}{n!}$$

ويجب أن تكون الدالة المراد إيجاد متسلسلة تايلر لها دالة مستمرة وقابلة للاشتقاق على فترة معينة حيث إن (n!) ونسمى بمضروب ال (n)

EX1:-

Find of the Tylor 's –series of f(x)=(x+2)^3 at a =2

$$f(a=2) = (2+2)^3 = 64$$

$$f'(x) = 3(x+2)^2 = f'(a=2) = 3(2+2)^2 = 48$$

$$f''(x) = 6(x+2) = f''(a=2) = 6(2+2)^1 = 24$$

$$f'''(x) = 6, stop$$

$$(x+2)^3 = 64 + \frac{48(x-2)^1}{1!} + \frac{24(x-2)^2}{2!} + \frac{6(x-2)^3}{3!}$$

$$= 64 + 48(x-2) + 12(x-2)^2 + (x-2)^3$$

EX2:-

Find of the Tylor 's –series of f(x)=e^x at a =1

$$f(a=1) = e^1 = e$$

$$f'(x) = e^x.1 = f'(a=1) = e^1 = e$$

$$f''(x) = e^x.1 = f''(a=1) = e^1 = e$$

$$f'''(x) = e^x.1 = f'''(a=1) = e^1 = e$$

:

:

$$f^n(x) = e^n.1 = f^n(a=1) = e^1 = e$$

$$e^x = e + \frac{e(x-1)^1}{1!} + \frac{e(x-1)^2}{2!} + \frac{e(x-1)^3}{3!} + ... + \frac{e(x-1)^n}{n!}$$

***Mclaren -series

سلسلة ماكلورين

إن سلسلة ماكلورين هي حالة خاصة من سلسلة تايلر وذلك عندما a =0 وان قانون سلسلة ماكلورين كالأتي:-

$$f(x) = S_n = f(0) + \frac{f'(0)(x)^1}{1!} + \frac{f''(0)(x)^2}{2!} + \frac{f'''(0)(x)^3}{3!} + ... + \frac{f^n(0)(x)^n}{n!}$$

EX1:-

Find of the Mclaren' –series of $f(x)=e^x$ at a =0

$$f(a=0) = e^0 = 1$$
$$f'(x) = e^x.1 = f'(a=0) = e^0 = 1$$
$$f''(x) = e^x.1 = f''(a=0) = e^0 = 1$$
$$f'''(x) = e^x.1 = f'''(a=0) = e^0 = 1$$

:

:

$$f^n(x) = e^n.1 = f^n(a=0) = e^0 = 1$$
$$e^x = 1 + \frac{1(x)^1}{1!} + \frac{1(x)^2}{2!} + \frac{1(x)^3}{3!} + ... + \frac{1(x)^n}{n!}$$
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!}$$

EX2:-

Find of the Mclaren' –series of f(x) =1/x at a =0

$$f(x) = \frac{1}{x}$$
$$f(a=0) = \frac{1}{0} = \infty$$
$$f'(x) = \frac{-1}{x^2} = \frac{-1}{(0)^2} = -\infty$$

The Mclaren –series dose not find

(9-4) اختبار التقارب للمتسلسلات موجبة الحدود

هنالك العديد من اختبارات التقارب للمتسلسلات

- اختبار الغاية.
- اختبار النسبة.
- اختبار الجذر.
- اختبار التكامل.
- اختبار المقارن.

(a9-4) اختبار النسبة Ratio-test

$$f(x) = \sum_{n=1}^{\infty} a_n \rightarrow positive$$

$$P = \lim_{n\to\infty} \frac{a_n + 1}{a_n}$$

$1 p \prec 1, convergent$

$2 p \succ 1, divergent$

$3 p = 1 fial$

EX1: By using Ratio-test of series convergent or divergent of

$$S_n = \sum_{n=0}^{\infty} \frac{2^n}{n}$$

$sol:-$

$$p = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}, a_n = \frac{2^n}{n}, a_{n+1} = \frac{2^{n+1}}{n+1}$$

$$p = \lim_{n\to\infty} \frac{\frac{2^{n+1}}{n+1}}{\frac{2^n}{n}} = \lim_{n\to\infty} \frac{2^{n+1}}{n+1} \cdot \frac{n}{2^n}$$

$$p = \lim_{n\to\infty} \frac{2^n . 2^1 . n}{2^n (n+1)}$$

$$p = \lim_{n\to\aleph} \frac{2n}{n+1} = \lim_{n\to\infty} \frac{\frac{2n}{n}}{\frac{n}{n} + \frac{1}{n}} = \lim_{n\to\infty} \frac{2}{1+\frac{1}{n}}$$

$$p = \frac{2}{1+\frac{1}{\infty}} = \frac{2}{1} = 2$$

$$p = 2 \succ 1, divergent$$

EX2: By using Ratio-test of series convergent or divergent of

$$S_n = \sum_{n=0}^{\infty} \frac{n+1}{3^n}$$

$sol:-$

$$p = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}, a_n = \frac{n+1}{3^n}, a_{n+1} = \frac{n+1+1}{3^{n+1}} = \frac{n+2}{3^{n+1}}$$

$$p = \lim_{n\to\infty} \frac{\frac{n+2}{3^{n+1}}}{\frac{n+1}{3^n}} = \lim_{n\to\infty} \frac{n+2}{3^{n+1}} \cdot \frac{3^n}{n+1}$$

$$p = \lim_{n\to\infty} \frac{3^n (n+2)}{3^n . 3(n+1)}$$

$$p = \frac{1}{3} \lim_{n\to\aleph} \frac{n+2}{n+1}$$

$$p = \frac{1}{3} = \lim_{n\to\infty} \frac{\frac{n}{n} + \frac{2}{n}}{\frac{n}{n} + \frac{1}{n}} = \frac{1}{3} \lim_{n\to\infty} \frac{1+\frac{2}{n}}{1+\frac{1}{n}}$$

$$p = \frac{1}{3} \prec 1, convergent$$

Root-test اختبار الجذر (b9-4)

يستخدم هذا الاختبار فقط للمتسلسلة التي تحمل الأس (n)

$$\text{P} = \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$$

الشروط

$1 p \prec 1, convergent$

$2 p \succ 1, divergent$

$3 p = 1 fial$

EX1: By using Root-test of series convergent or divergent of

$$S_n = \sum_{n=0}^{\infty} \frac{1}{(n)^n}$$

$$S_n = \sum_{n=0}^{\infty} \frac{1}{(n)^n} = \sum_{ى=0}^{\infty} (\frac{1}{n})^n = \lim_{n\to\infty} \sqrt[n]{(\frac{1}{n})^n} = ((\frac{1}{n})^n)^{\frac{1}{n}}$$

$$\lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$$

$$p = 0 \prec 1, convergent$$

EX2: By using Root-test of series convergent or divergent of

$$S_n = \sum_{n=0}^{\infty} \frac{1}{(n(n+1))^n}$$

$$S_n = \sum_{n=0}^{\infty} \frac{1}{(n(n+1))^n} = \sum_{n=0}^{\infty} \left(\frac{1}{n(n+1)} \right)^n =$$

$$p = \lim_{n\to\infty} \sqrt[n]{\left(\frac{1}{n^2+n} \right)^n} = lin_{n\to\infty} \left(\left(\frac{1}{n^2+n} \right)^n \right)^{\frac{1}{n}} =$$

$$p = \lim_{n\to\infty} \frac{1}{n^2+n}$$

$$p = \lim_{n\to\infty} \frac{\frac{1}{n^2}}{\frac{n^2}{n^2} + \frac{n}{n^2}} = \lim_{n\to\infty} \frac{\frac{1}{n^2}}{1+\frac{1}{n}} = \frac{\frac{1}{(\infty)^2}}{1+\frac{1}{\infty}}$$

$$p = \frac{0}{1+0} = 0$$

$$p = 0 \prec 1, convergent$$

الواجبات

Q1/ Find the sequence and draw the sequence

$1/ a_n = \{(-1)^n\}_{n=1}^{4}$

$2/ a_n = \{e^n\}_{n=1}^{\infty}$

Q2/ If the sequence defined by the functions of the sequences

$1/ a_n = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\right\}$

$2/ a_n = \{1, -1, 1, -1, 1, -1, ...\}$

Q3/ Is the sequence is convergent or divergent

$$1/\, a_n = \frac{n^2 + 2n}{n^2 + 1}$$

$$2/\, a_n = \frac{1}{n^2}$$

Q4/ Find the convergent or divergent , bounded or un bounded , incensing or decreasing of the sequence

$$1/\, a_n = \sin(\frac{1}{n})$$

$$2/\, a_n = \sin(n)$$

$$3/\, a_n = n + 1$$

$$4/\, a_n = \frac{n}{2}$$

Q5/ Find the convergent or divergent of series

$$1/\, S_n = \sum_{n=1}^{\infty} 1 + \frac{1}{n}$$

$$2/\, S_n = \sum_{n=1}^{\infty} \frac{n^3 + 5n}{n^4 - 6}$$

$$3/\, S_n = \sum_{n=0}^{\infty} \frac{n^2 - 5}{n + 1}$$

Q6/ Find (n th) partial sum and (n th) term of the series

$$1/\, S_n = \{3 + 8 + 13 + 18 + 23 + 28 + ...\}$$

$$2/\, S_n = \left\{6 + 2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + ...\right\}$$

Q7/ Find the Tylors-series of $f(x)=x^2+4x$ at $a=1$

Q8/ Find the Maclourn –series of

$$1/\, f(x) = \sin(x)$$

$$2/\, f(x) = \cos(x)$$

Q9/ By using the Ratio-test find convergent or divergent

$$1/ \sum_{n=0}^{\infty} \frac{4^n}{n^2}$$

$$2/ \sum_{n=0}^{\infty} -e^x$$

Q10/ By using the Root-test find convergent or divergent

$$\sum_{n=1}^{\infty}\left(\frac{1}{|x|}\right)^{n}$$

Q11/ Find the value of (c) that satisfy Roll theorem of $f(x) = x^2+3x-1$; [-3,-1]